

# A Health Monitoring Method for Li-ion Batteries Based on Profust Reliability Theory

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This paper presents an applicable health monitoring method for Li-ion batteries based on profust reliability theory. The word “profust” derives from “probability” and “fuzzy-state”, which means that the profust reliability theory is a kind of fuzzy reliability theory based on probability assumption and fuzzy-state assumption. It defines a concept of transition from fuzzy success to fuzzy failure to characterize system performance. In this paper, charge cycles of a battery are analysed, and the profust reliability is employed to estimate battery health state as a health indicator. Then, all operational states are classified into five health levels through profust reliability values. This process contributes a transform from quantitative reliability index to qualitative health level, which is able to describe system performance in a more intuitive way. Owing to the appealing ‘fuzzy’ advantage, the profust reliability theory can be effectively applied to the health monitoring of Li-ion batteries.

## 1. Introduction

Prognostics and health management (PHM) is defined as a health management approach utilizing measurements, models, and software to perform incipient fault detection, condition assessment, and failure progression prediction (Kalgren et al., 2006). Activities of PHM date back to engine monitoring system of A-7E aircraft in 1970s. Currently, there are PHM methods for many different types of vehicles, systems, and products from electric components (Pecht and Gu, 2009) to the Joint Strike Fighter (Hess and Fila, 2002).

Battery is a core component of various complex systems, which has a great influence on the performance of overall system (Goebel et al., 2008). Failure of a battery can lead to unsatisfied performance, unexpected halting problem, and even catastrophic failure. In this case, health monitoring of batteries is extremely considerable, which is able to estimate the battery operational state, and schedule operation strategies to guarantee maximum reliability, safety and economical efficiency (Saha et al., 2009a). There have already existed a large number of research focusing on battery health management (Saha et al., 2009b). This paper proposes a novel health monitoring method for Li-ion batteries based on profust reliability theory (Cai et al., 1991, 1996). In the profust reliability theory, the system performance is described in fuzzy-states rather than binary states, and the profust reliability can be employed to measure real-time system performance through transitions among system operational states. Thus, it is more reasonable to use profust reliability theory to perform health monitoring. In this paper, the charge cycle of batteries is analysed, and real-time continuous charge capacity signal is obtained and used as a monitored parameter. Then, the profust reliability is calculated through state discretization of the continuous charge capacity signal, which characterizes the battery health state as a health indicator, and then ranks all health states into different health levels. In future research, remaining useful life (RUL) prediction and operation strategy optimization will be performed based on the established health levels.

## 2. Analysis of Battery Charge Cycles

The data of Li-ion batteries used here comes from NASA Ames prognostics data repository (Saha and Goebel, 2007). The battery data set records accelerated aging experiments of different batteries, which

are run through repeated charge and discharge cycles at different ambient temperature. Charging is carried out in a constant current (CC) mode at 1.5 A until the battery voltage reaches 4.2 V and then continued in a constant voltage (CV) mode until the charge current drops to 20 mA. Discharging is carried out at a CC level of specific values until the battery voltage falls to a specific value for specific batteries respectively. The experiments are stopped until the battery charge retention capacity drops to 70 % of its full capacity observed at the beginning. Here, we mainly concentrate on the charge cycles. The battery health state of each charge cycle is estimated, and the remaining charge cycle until the battery reaches EOL criteria is predicted using the proposed method.

By taking battery #5 as an example, the data contains 167 whole charge cycles. As mentioned before, charge is carried out in a CC mode at 1.5 A until the battery voltage reaches 4.2 V and then continued in a CV mode until the charge current drops to 20 mA. For ease of visualization, the variations of charge voltage and current of 8 charge cycles are shown in Figure 1.

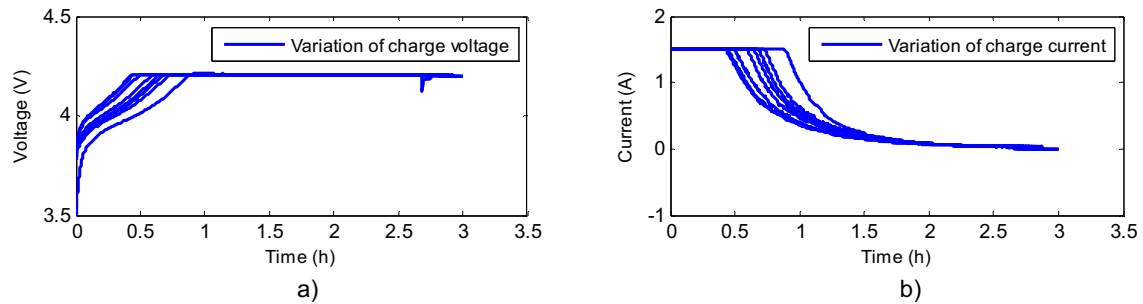


Figure 1: a) Charge voltage variation, b) Charge current variation

Here, let  $u(t)$  and  $i(t)$  represent the corresponding voltage variable and current variable, respectively. In order to consider both  $u(t)$  and  $i(t)$ , battery charge capacity is used as an assessable index in battery health estimation, which is defined in Eq.(1)

$$W(t) = \int_0^t u(\tau) \cdot i(\tau) d\tau \quad (1)$$

where  $W(t)$  represents the charge capacity, and the unit is VAh. Then, the real-time charge capacity of all 167 charge cycles is calculated using Eq.(1). Similarly, the real-time charge capacity of the preceding 8 charge cycles are shown in Figure 2a.

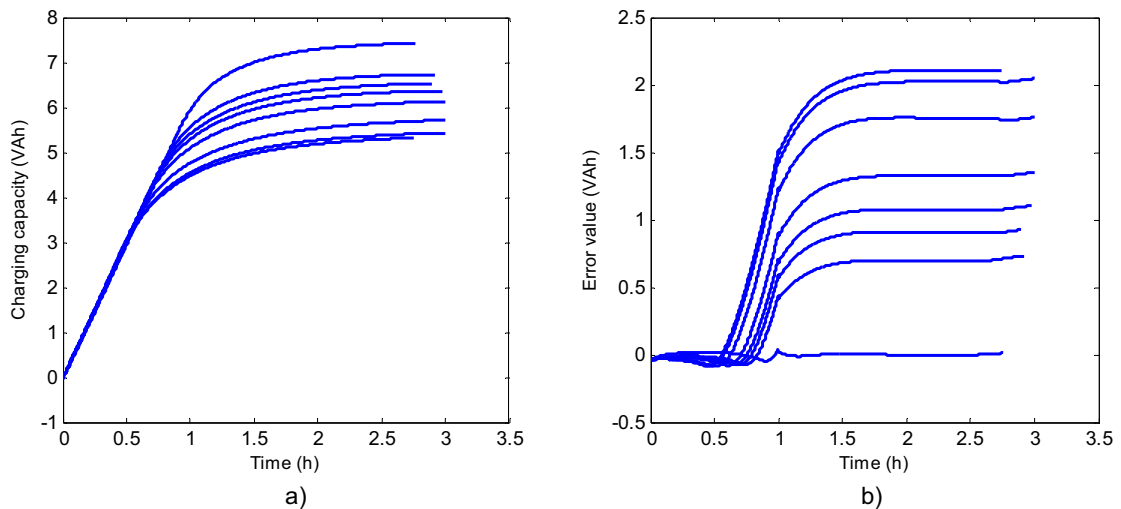


Figure 2: a) Charge capacity variation, b) Error of charge capacity deviated from the standard value

As we know, the first charge cycle after fully discharging is able to charge the battery to the maximum power, which corresponds to the highest curve in Figure 2a. In this case, we consider it as the fully healthy state of battery charge process, which also can be viewed as a standard charge cycle. Furthermore, we

consider that the battery starts to degrade after the first charge cycle. Then, let  $e(t)$  represent the error that real-time variation of charge capacity deviates from the proposed standard curve of charge capacity variation. Thus, we have

$$e_{\Delta_k}(t) = W_{\Delta_1}(t) - W_{\Delta_k}(t), \quad k = 2, 3, \dots, 167 \quad (2)$$

where  $W_{\Delta_1}(t)$  and  $W_{\Delta_k}(t)$  are the real-time charge capacity at charge cycle  $\Delta_1$  and  $\Delta_k$ , respectively. Obviously, for each charge cycle,  $e(t)$  is a continuous signal as shown in Figure 2b.

### 3. The Health Monitoring Method for Li-ion Batteries Based on Profust Reliability Theory

#### 3.1 Theoretical part of the proposed method

In this section, the profust reliability theory is used to monitor the performance of Li-ion batteries, where profust reliability value is used to evaluate the battery health state as a health indicator. In order to calculate profust reliability value proposed in (Cai et al., 1996), the continuous signal  $e(t)$  obtained above is supposed to be discretized into different health states. Here, let

$$S_i = \{e(t) \mid a_{i-1} \leq e(t) \leq a_i\} \quad (3)$$

where

$$a_0 = -2, a_n = 2, \Delta_n = \frac{a_n - a_0}{n}; a_i = a_0 + i \cdot \Delta_n, i = 1, 2, \dots, n \quad (4)$$

$$S_{n+1} = \{e(t) \mid e(t) < a_0 \text{ or } e(t) > a_n\} \quad (5)$$

Then, the battery has discrete states  $S_1, \dots, S_{n+1}$ . Let  $U = \{S_1, \dots, S_{n+1}\}$  be the domain of discourse. In the domain  $U$ , system fuzzy success states can be defined as:

$$S = \{S_i, \mu_S(S_i); i = 1, 2, \dots, n+1\} \quad (6)$$

and fuzzy failure states

$$F = \{S_i, \mu_F(S_i); i = 1, 2, \dots, n+1\} \quad (7)$$

where  $\mu_S(S_i)$  and  $\mu_F(S_i)$  are the corresponding membership functions, respectively.

Let  $U_T = \{m_{ij}; i = 1, \dots, n+1, j = 1, \dots, n+1\}$ , where  $m_{ij}$  represents the transition from state  $S_i$  to state  $S_j$ . In the domain  $U_T$ , a transition from fuzzy success state to fuzzy failure state is defined as (Cai et al., 1996):

$$T_{SF} = \{m_{ij}, \mu_{T_{SF}}(m_{ij}); i, j = 1, 2, \dots, n+1\} \quad (8)$$

Here,  $T_{SF}$  is viewed as a fuzzy event, and the corresponding membership function  $\mu_{T_{SF}}(m_{ij})$  is determined as (Cai et al., 1996):

$$\mu_{T_{SF}}(m_{ij}) = \begin{cases} \beta_{F|S}(S_j) - \beta_{F|S}(S_i) & \text{if } \beta_{F|S}(S_j) > \beta_{F|S}(S_i) \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

$$\beta_{F|S}(S_i) = \frac{\mu_F(S_i)}{\mu_S(S_i) + \mu_F(S_i)}; i = 1, 2, \dots, n+1 \quad (10)$$

Let  $\Delta_k$  represent the  $k$ th charge cycle. The profust reliability at charge cycle  $\Delta_k$  can be defined as (Cai et al., 1996):

$$R(\Delta_k) = P\{T_{SF} \text{ does not occur during time interval } [\Delta_{k-1}, \Delta_k]\} \quad (11)$$

Eq.(11) only takes transition process during the charge cycle  $\Delta_k$  into consideration. However, when the system state at charge cycle  $\Delta_{k-1}$  is more fuzzy failed than fuzzy successful, the initial system state at charge cycle  $\Delta_{k-1}$  should not be ignored any more. For example, assuming that the system has already stayed in full failure state at charge cycle  $\Delta_{k-1}$ , the value of  $R(\Delta_k)$  should be equal to 0. However,  $T_{SF}$  does not occur during the charge cycle  $\Delta_k$  indeed, which means that  $R(\Delta_k)$  is equal to 1 obtained through Eq.(11). Thus, when the system starts to degrade in the process of PHM, it is inappropriate to use

Eq.(11) to calculate profust reliability all the time. For such a purpose, a new algorithm of profust reliability is proposed here.

Define fuzzy event:

$$A = \{T_{SF} \text{ does not occur during the charging cycle } \Delta_k\}$$

$$B = \{\text{The battery is in fuzzy success state at charging cycle } \Delta_{k-1}\}$$

Here, system state at charge cycle  $\Delta_{k-1}$  is taken into consideration. Then, the profust reliability at charge cycle  $\Delta_k$  can be defined as:

$$\begin{aligned} R(\Delta_k) &= P(A|B) \cdot P(B) \\ &= 1 - P(\bar{A}|B) \cdot P(B) - P(\bar{B}) \\ &= 1 - \left[ \sum_{i=1}^{n+1} \sum_{j=1}^{n+1} \mu_{T_{SF}}(m_{ij}) \cdot p_{ij}(\Delta_k) \right] \cdot \left[ \sum_{i=1}^{n+1} \mu_S(S_i) \cdot \phi_{S_i}(\Delta_{k-1}) \right] - \sum_{i=1}^{n+1} \mu_F(S_i) \cdot \phi_{S_i}(\Delta_{k-1}) \end{aligned} \quad (12)$$

where  $p_{ij}(\Delta_k)$  represents the probability of transition from state  $S_i$  to state  $S_j$  at charge cycle  $\Delta_k$ , and  $\mu_{T_{SF}}(m_{ij})$  represents the corresponding membership function;  $\phi_{S_i}(\Delta_{k-1})$  represents the probability of system state  $S_i$  at charge cycle  $\Delta_{k-1}$ , and  $\mu_S(S_i)$  and  $\mu_F(S_i)$  represent the corresponding fuzzy success and failure membership functions, respectively.

Because the health state of complex system cannot be just defined as "successful" and "failed" through single threshold segmentation, system states should be classified into different health levels in terms of the range of  $R(\Delta_k)$ . Here, we have

$$\text{System Health Level} = \begin{cases} \text{Health Level-1} & R \in [0, 0.4) \\ \text{Health Level-2} & R \in [0.4, 0.5) \\ \text{Health Level-3} & R \in [0.5, 0.6) \\ \text{Health Level-4} & R \in [0.6, 0.8) \\ \text{Health Level-5} & R \in [0.8, 1) \end{cases} \quad (13)$$

In this case, all operational states of a battery can be classified into 5 health levels. This process contributes a transform from a quantitative reliability index to qualitative health levels, which can describe system performance in a more intuitive way.

### 3.2 Algorithm Implementation and simulation results

In order to calculate the profust reliability  $R(\Delta_k)$  of each charge cycle through Eq.(12), two steps are supposed to be implemented.

1) Calculation of fuzzy success/failure membership function

Under the fuzzy-state assumption, the operational state of charge capacity characterizing Li-ion batteries can be described with fuzzy health membership of the continuous signal  $e(t)$ . In this case, appropriate selection of fuzzy health membership is critical to health monitoring accuracy. Here, according to battery prior knowledge, experiment data, and expert experience, we have

$$\mu_s(e(t), -2, -1, 1, 2) = \begin{cases} \frac{e(t)+2}{1} & e(t) \in (-2, -1] \\ 1 & e(t) \in (-1, 1] \\ \frac{e(t)-2}{-1} & e(t) \in (1, 2] \\ 0 & e(t) \in (-\infty, -2] \cup (2, +\infty) \end{cases} \quad (14)$$

and

$$\mu_f(e(t), -2, -1, 1, 2) = 1 - \mu_s(e(t), -2, -1, 1, 2) \quad (15)$$

Then, for discrete state  $S_i$  ( $i = 1, \dots, n$ ), we have

$$\mu_S(S_i) = \frac{1}{\Delta_n} \int_{a_{i-1}}^{a_i} \mu_s(e(t)) de(t) \quad (16)$$

$$\mu_F(S_i) = \frac{1}{\Delta_n} \int_{a_{i-1}}^{a_i} u_F(e(t)) de(t) \quad (17)$$

Furthermore, we can obtain  $\mu_S(S_{n+1}) = 0, \mu_F(S_{n+1}) = 1$  through Eq.(14).

2) Calculation of transition probability and state probability

Let  $P(\Delta_k) = (p_{ij}(\Delta_k))_{(n+1) \times (n+1)}$  represent the transition probability matrix of battery states at charge cycle  $\Delta_k$ ,

where  $p_{ij}(\Delta_k)$  represents the probability of transition from state  $S_i$  to state  $S_j$ . The value of  $p_{ij}(\Delta_k)$  can be estimated through maximum likelihood estimation (MLE),

$$p_{ij}(\Delta_k) = \frac{n_{ij}(t_0^{(k)}, t_n^{(k)})}{\sum_{j=1}^{n+1} n_{ij}(t_0^{(k)}, t_n^{(k)})} \quad (18)$$

where  $n_{ij}(t_0^{(k)}, t_n^{(k)})$  represents the number of transition from state  $S_i$  to state  $S_j$  in time interval  $[t_0^{(k)}, t_n^{(k)}]$ ;

$t_0^{(k)}$  represents the start time of charge cycle  $\Delta_k$ , and  $t_n^{(k)}$  represents the end time. Let

$\Phi(\Delta_k) = (\phi_{S_i}(\Delta_k))_{1 \times (n+1)}$  represent the state probability vector at charge cycle  $\Delta_k$ . After obtaining  $P(\Delta_k)$ , we

have

$$\Phi(\Delta_k) = \Phi(\Delta_{k-1}) \cdot P(\Delta_k) = \dots = \Phi(\Delta_0) \cdot \prod_{i=1}^k P(\Delta_i) \quad (19)$$

where  $\Phi(\Delta_0)$  represents the state probability vector of the first charge cycle. Assume that state  $S_h$  in domain  $U$  is the fully healthy state. In this case, for  $\Phi(\Delta_0)$ , we have

$$\begin{cases} \phi_{S_h} = 1 \\ \phi_{S_i} = 0; i = 1, \dots, n+1 \text{ and } i \neq h \end{cases} \quad (20)$$

From the above, using obtained voltage and current signals from accelerated aging experiments, the profust reliability  $R(\Delta_k)$  and the health levels of all 167 charge cycles can be calculated through Eq.(12) and Eq.(13). Up to now, the health monitoring of Li-ion batteries is achieved, and the result is shown in Figure 3.

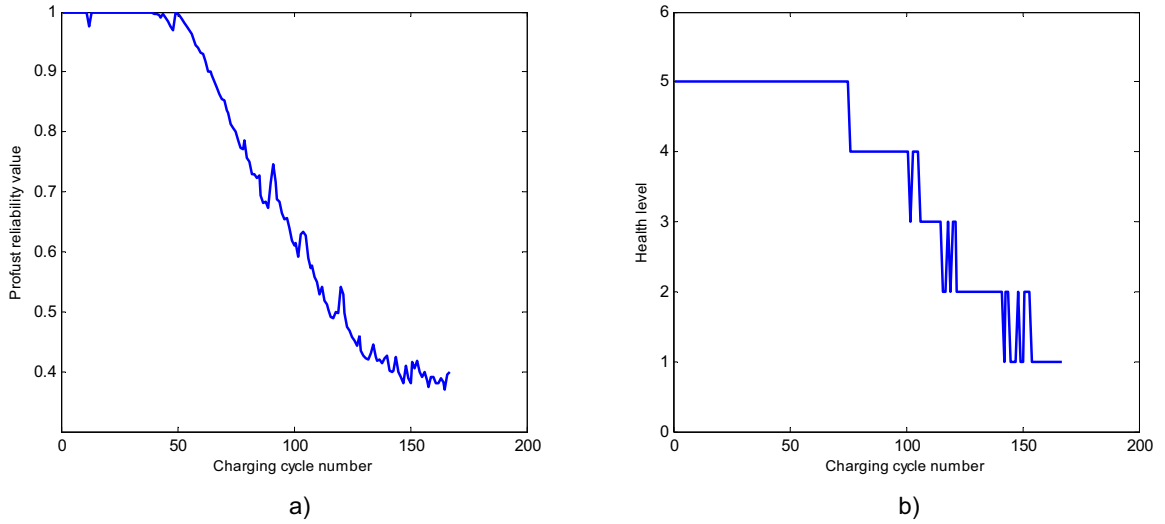


Figure 3: Health monitoring result of all 167 charge cycles a) profust reliability value, b) Health level variation

#### 4. Conclusion

This paper proposes a health monitoring method for Li-ion batteries based on profust reliability. Charge cycles of batteries are analysed here, and the profust reliability value is used to estimate battery health state as a health index. Then, all operational states are ranked into five health levels using profust reliability values. In future research, the remaining useful life or remaining charge cycles of Li-ion batteries should be predicted based on the proposed health levels. Furthermore, the proposed health monitoring method can be applied to other products and systems, and a combination between prognostics and health management (PHM) and profust reliability theory is supposed to be further implemented.

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