

## Maintenance Optimisation of Optrononic Equipment

Camille Baysse<sup>\*a,b</sup>, Didier Bihannic<sup>b</sup>, Anne Gegout-Petit<sup>a</sup>, Michel Prenat<sup>b</sup>,  
Benoite de Saporta<sup>a</sup>, Jérôme Saracco<sup>a</sup>

<sup>a</sup>Thales Optronique, 2 Avenue Gay Lussac, 78990 Élanecourt France.

<sup>b</sup>Bordeaux University, UMR CNRS 5251 IMB, INRIA Bordeaux Sud-ouest CQFD, 200, avenue de la Vieille Tour 33405 Talence cedex France.  
[camille.baysse@inria.fr](mailto:camille.baysse@inria.fr)

As part of optimizing the reliability, Thales Optronics now includes systems that examine the state of its equipment. This function is performed by HUMS (Health & Usage Monitoring System). The aim is to implement in the HUMS a program based on observations that can determine the state of the system and propose a maintenance action before failures. So we decompose our problem into two steps: the first step is to detect the degraded state (which announces future failure) using an informative variable and hidden Markov chains. This step was developed in Baysse et al. (2012). The second is to propose an optimal and dynamic maintenance policy, adapted to the state of the system and taking into account both random failures and those related to the degradation phenomenon. We want to estimate the best time to perform maintenance: a maintenance performed too early may be unnecessarily costly and inconvenient for the client but too late may cause the occurrence of a failure that will damage the rest of the equipment and may be responsible for the failure of a mission. So it is necessary to find a balance between these two extreme maintenance policies. First, we model the state of the system by a piecewise-deterministic Markov process: PDMP introduced by Davis (1993). Often the evolution of the system is modeled by stochastic processes such as Markov jump process, semi-Markov process (Cocozza et al. (1997)). There are also tools for modeling such as Stochastic Petri networks (Marsan et al. 1995), dynamic Bayesian networks (Donat et al. 2010). However, the flexibility of modeling by PDMP allows to take into account the dynamic component degradation. The works of Lair et al. (2012) focuses on this topic, they use a finite volume scheme to evaluate the quantities of interest associated with PDMP. Even if there are different methods that optimize maintenance policy, few use optimal stopping. In this paper, we use this method whose principle is to maximize a performance function that takes into account operating time, maintenance costs, repairs and downtime. We use the numerical probability tools developed in de Saporta et al. (2012) in order to compute this conditioned-based time of maintenance. The integration of this method in the HUMS, will be soon implemented in specific optrononic equipment by Thales. We present results of simulation in this case. The methodology can be extended to more complicated cases.

### 1. Industrial context

Thanks to the HUMS, each of the appliances has a logbook which provides information at each start-up such as: number of uses, cumulative operating time of appliance, "cool down time" (T<sub>mf</sub>)... This T<sub>mf</sub> is the transit time for the system from ambient temperature to a very low one. This temperature decrease is required to operate appliance and this is done on every boot. According to experts, a T<sub>mf</sub> increase results from deterioration in the cooling system. According to this hypothesis, a careful observation of T<sub>mf</sub> evolution allows us to determine the state of the cooling system. We suppose that the cooling system pass from stable state to degraded state and from degraded state to reach failure. In Baysse et al. (2012), we have given a mathematical method based on Hidden Markov Chain in order to detect a transition to a degraded state of the cooling system. There are two other kinds of possible failures: electronic failure and ball bearing failure. These two failures do not pass by a degraded state. So we study appliance with three failures (electronic, ball bearing and cooling system failure) and three states (stable, degraded on account of cooling system, failure).

Our objective is to propose an optimal and dynamic maintenance policy adapted to the random state of the system.

## 2. Modeling

In order to develop a maintenance policy that takes into account both random failures and those related to a degradation phenomenon, we model the state of the system by a piecewise-deterministic Markov process. This modeling makes possible the transition from the stable state to failure directly (random case) or through the degraded state (damage to the cooling system (see Figure 1)).

The notion of PDMP was introduced by Davis (1993). PDMP are processes with deterministic evolution, punctuated by random jumps and changes of regimes that can allow them to pass from one state to another. PDMP are hybrid process generally noted  $\xi_t = (m_t, s_t)_{t \in \mathbb{R}_+}$ . The first component  $m_t$  is a discrete variable with values in a finite or countable space  $M$ . It describes the state of the system at time  $t$  (system in stable mode, degraded, failure ...). The second component  $s_t$  evolves in a continuous way in  $E_m \in \mathbb{R}^n$  and describes evolution of the system in the mode  $m_t$  by its physical variables (for example pressure, age of system..).

Our study is about equipment with three states: stable state ( $m_t = 1$ ), degraded state ( $m_t = 2$ ) and failure ( $m_t = 3$ ).

At the beginning equipment is in stable state and then it breakdowns or it goes in degraded state:

- If it breakdowns directly, it is due to an electronic failure or a failure about ball bearing. Failure rates are respectively  $\lambda_1$  and  $\lambda_3(t)$ .
- If it goes from stable state to a degraded state, it is due to a deterioration of the cooling system. The occurrence rate of this deterioration is noted  $\lambda_0$ . In the degraded state, it is possible to have electronic, ball bearing or cooling system failures (rate  $\lambda_2$ ).

Equipment in degraded state or in failure cannot return in stable state. Note that the state "failure" is absorbant and so the number of jump is less than or equal to 2. Figure 1 illustrates how the system works. According to experts, the rate  $\lambda_3$  depends on the age  $t$  of equipment contrary to other rates. Note that  $m_t$  is not markovian because rate  $\lambda_3$  depends on  $t$ . Thanks to our method to detect transition from state 1 to state 2, (see Baysse & al (2012)), we suppose that the jump of the process is observed.

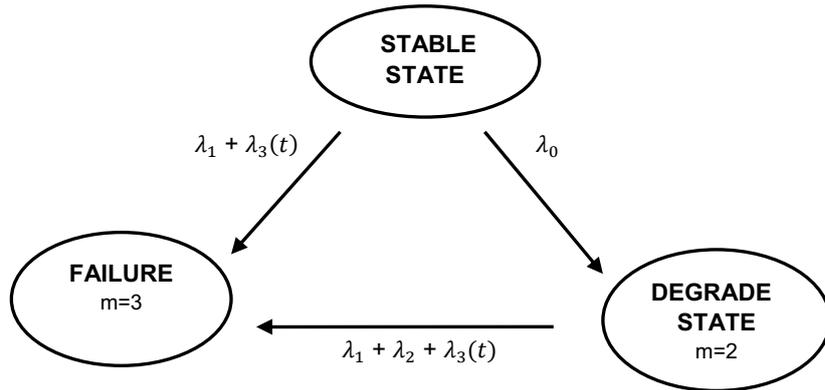


Figure 1 : Modeling system

In order to use the powerful framework of Markov process, we must add time  $t$  to the process  $(m_t)_{t \in \mathbb{R}_+}$  as information such that  $\xi_t = (m_t, s_t = t)_{t \in \mathbb{R}_+}$  is markovian. Indeed it is a PDMP. So the PDMP considered here describes the state of equipment and its age:  $\xi_t = (m_t, s_t = t)_t$ . Its motion is described by the three characteristics (see Davis (1993)):

- the flow  $\phi(m, t; s) = (m, t+s)$ ,
- the rate of jump  $\lambda(m_t, t) = (\lambda_0 + \lambda_1 + \lambda_3(t))1_{\{m_t=1\}} + (\lambda_1 + \lambda_2 + \lambda_3(t))1_{\{m_t=2\}}$ ,
- the measure of transition:

$$Q(m_t, t; \{e\} \times \{t\}) = \begin{cases} \left( \frac{\lambda_0}{\lambda_0 + \lambda_1 + \lambda_3(t)} \right) 1_{\{e=2\}} + \left( \frac{\lambda_1 + \lambda_3(t)}{\lambda_0 + \lambda_1 + \lambda_3(t)} \right) 1_{\{e=3\}} & \text{if } m_t = 1 \\ 1_{\{e=3\}} & \text{if } m_t = 2 \end{cases} \quad (1)$$

We denote  $Z_0 = (1, 0)$ ,  $T_0 = 0$ . The state of the system just after the first transition is  $Z_1 = (m_{T_1}, T_1)$  and  $Z_2 = (m_{T_2}, T_2) = (3, T_2)$  if  $T_2$  occurs. We put  $S_1 = T_1$  and  $S_2 = T_2 - T_1$  the interjumping times. With these notations, the discrete process  $\theta_n = (Z_n, S_n) = (m_{T_n}, T_n, S_n)$  associated with the PDMP  $(\xi_t)_t$  is a Markov chain and it is considered for  $n \in \{0, 1, 2\}$ .

The horizon  $T$  of the study is finite. So that the remaining time is  $t^*(\xi_t) = T - t$ .

### 3. Optimal and dynamic maintenance policy

We consider the problem as an optimal stopping problem for PDMP, whose principle is to maximize a performance function that takes into account operating time, maintenance costs, repairs and downtime. We want to estimate the best time to perform maintenance in order to allow Thales to manage upstream park equipment. Recent work has been done on this subject. De Saporta et al. (2012) give the theoretical foundations of the method that we use in this study. In de Saporta et al. (2010), a method of computation of best time to perform maintenance on a complex dynamic system is implemented and analyzed.

#### 3.1 Principle of optimal stopping time

Our aim is to find a stopping time  $\tau$  which maximizes expectation of a performance function at random stopping time  $\tau$  that is  $E_{\xi_0=(m_0,0)}[g(m_\tau, \tau)]$  where  $g$  is the function of system performance and  $\tau$  a stopping time adapted to the filtration of the PDMP. This problem is typically an optimal stopping problem which consists in solving the following optimization problem:

$$v_0(m_0, 0) = \sup_{\tau \leq t^*(m_0,0)} E_{\xi_0=(m_0,0)}[g(m_\tau, \tau)] \quad (2)$$

with  $\begin{cases} v_2 = g \\ v_1 = L(v_2, g) \end{cases}$

Function  $v_0$  is called the value function of the problem and represents the maximum performance that can be achieved. Operator  $L$  is defined by  $L(w, g)(x) = \sup_{\tau \leq t^*(x)} \{E_x[w(Z_1)1_{\{S_1 < \tau\}}] + g(\phi(x; \tau))P_x(S_1 \geq \tau)\} E_x[w(Z_1)]$ . It is a complex operator that depends on the characteristics of the PDMP. However, we can see that it depends on the PDMP only through the underlying Markov chain  $\theta_n$ . In our case, we chose the performance function  $g(m_t, t) = \begin{cases} t & \text{if } m_t = 1 \text{ or } 2 \\ 0 & \text{if } m_t = 3 \end{cases}$ . Here this function favours a long time of use but is canceled if the system fails. In practice, the optimal stopping time does not necessarily exist. However, we can always find time to stop that approach the optimal performance as near as you want.

#### 3.2 Numerical method of optimal stopping

We apply methodology developed in de Saporta et al. (2010).

To approximate the  $\varepsilon$ -optimal stopping time  $\tau$  we introduce a sequence of random variables  $(V_n)_{n \in \{0,1,2\}}$  such as  $V_n = v_n(Z_n)$ . This allows to replace the recurrence (2) which covers functions by a recurrence on random variables easier to treat numerically. To approximate the values of this sequence, we proceed in two steps. First, we discretize the process on a regular time grid noted  $G(\xi)$  associated with interval  $[0, t^*(\xi)]$ , in order to obtain a discrete time Markov chain. Thus the operator  $L$  is maximized on a finite number of points and not on a continuous time interval. This new discretized operator is noted  $L^d$ . The second step is the quantization that transforms the continuous random variables  $\theta_n$  into a discrete random variable  $\hat{\theta}_n$ . Quantization provides a finite set of points (a grid) adapted to the law of the process and not arbitrary regular basis on the state space. Details of this method are given by Pagès et al. (2003). It is based on simulations of the Markov chain  $(\theta_n)$ . So we denote  $\hat{\theta}_n = (\hat{Z}_n, \hat{S}_n) = (\hat{m}_n, \hat{T}_n, \hat{S}_n)$  projection of  $\theta_n$  on the quantization grid  $\Gamma_n^Z$ . After these two steps, the operator  $L$  is approximated by operators  $\widehat{L}_k^d$  for  $k \in \{1,2\}$ .

Now we can build a sequence of variables  $(\widehat{V}_n)$  which approaches  $(V_n)$ . To do this, we first consider the following process:

$$\begin{cases} \hat{v}_2(z) = g(z) \text{ with } z \in \Gamma_2^Z, \\ \hat{v}_1(z) = \widehat{L}_2^d(\hat{v}_2, g)(z) \text{ with } z \in \Gamma_1^Z, \\ \hat{v}_0(z) = \widehat{L}_1^d(\hat{v}_1, g)(z) \text{ with } z \in \Gamma_0^Z. \end{cases} \quad (3)$$

$$\text{with } \widehat{L}_k^d(w, g)(\xi) = (\max_{t \in G(\xi)} \{E[w(\hat{Z}_k)1_{\{\hat{S}_k < t\}} | b_{k-1}^\xi] + g(\phi(\xi; t))P(\hat{S}_k \geq t | b_{k-1}^\xi)\}) E[w(\hat{Z}_k) | b_{k-1}^\xi]. \quad (4)$$

Where  $b_{k-1}^\xi$  is the event  $\hat{Z}_{k-1} = \xi$ .

The approximation of  $V_k$  is performed by  $\widehat{V}_k = \hat{v}_k(\hat{Z}_k)$  for  $k \in \{0,1,2\}$ . It is shown in de Saporta et al. (2010) that the error of approximation of the value function  $|\widehat{V}_0 - V_0|$  can be made arbitrarily small by a suitable choice of discretization parameters). A stopping time arbitrarily close to the optimal is also provided.

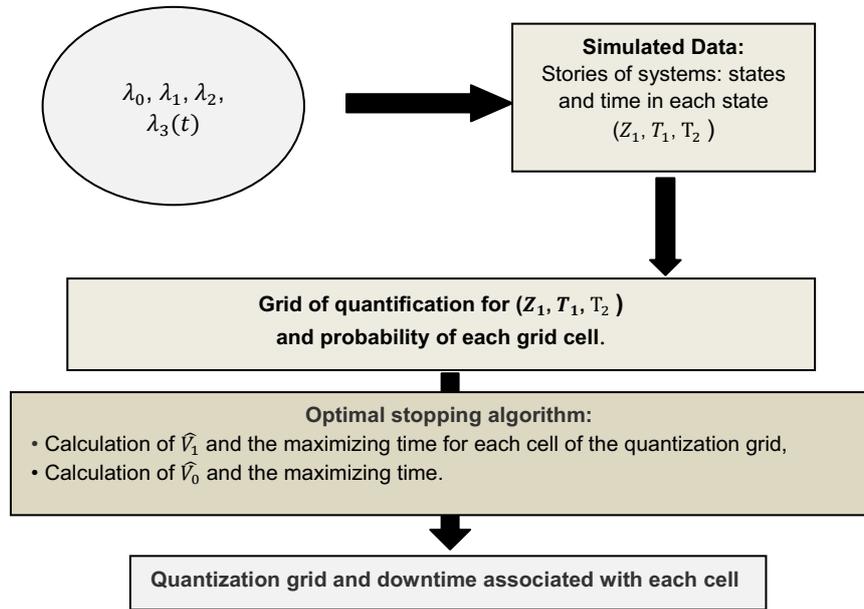


Figure 2: Schematic representation of the algorithm

#### 4. Applications

A general presentation of the algorithm used to perform the maintenance policy is given in Figure 2. From the value of  $T$  and failure rates  $\lambda_0, \lambda_1, \lambda_2, \lambda_3(t)$  provided by experts, we built a simulator of the trajectories of the process. For each of them, we have the following information: the time  $S_1$  spent in the stable state, the type of jump ( $m_{T_1}=2$  or 3) and the time spent in the new state if  $m_{T_1} = 2$ . From this simulation, we have created the quantization grid using an algorithm given in Pagès et al. (2003). All of these elements will allow us to calculate  $\hat{V}_0$  and  $\hat{V}_1$  of each cell and the times  $\tau_0, \tau_1$  which maximize them. Note that  $\tau_0$  is deterministic and  $\tau_1$  depends on the cell. So for each cell of the quantization grid we can associate a time nearly optimal. Let us remark that the result of this algorithm only depends on  $T$  and the failure rates and it is compute once and for all.

It will suffice to project data of equipment chosen on the new grid to propose stopping time that will be associated.

In the practice, maintenance policy is the following:

- at the beginning, a maintenance date is announced at a fixed date  $\tau_0$  for all equipment,
- if an appliance goes in degraded state to the time  $T_1$  before the date fixed  $\tau_0$ , maintenance time is recalculated and replaced by a new time  $\tau_1$ . The time  $\tau_1$  is given by the optimal stopping time associated to the cell of  $\hat{T}_1$  (the projection of  $T_1$  on the grid).

To illustrate this point, we choose to look at the history of 10 appliances. In parallel we launched the algorithm to build the downtime for each device. Examples of results are presented in Table 1.

Table 1: Results of simulations

Equipment (n°)	1	2	3	4	5	6	7	8	9	10
$T_1$	6899	3766	6802	2238	7090	3432	4162	3800	4212	2579
$m_{T_1}$	2	2	3	2	3	3	2	2	2	2
$T_2$ if $Z_1 = (2, T_1)$	6981	3834	-	2598	-	-	4309	3885	4393	2627
Maintenance date	5160	3827	5160	2508	5160	3432	4192	3860	4242	2627

We have three possible cases:

- maintenance is at time  $\tau_0$  and before the first jump (ex n°1,3,5). We can remark that in cases 3 and 5, the first jump would have resulted to a failure.

- maintenance is between the first and second jump, when the system is in a degraded  $m_{T_1} = 2$  (cases n°2,4,7,8,9). Maintenance is also before failure.
- maintenance is triggered by the failure of the system (cases n°6,10). Indeed, algorithm had planned to stop equipment n° 6 at time 5160 and n° 10 at around 2679 but these two appliances broke down before this date (that is why the stopping time equals downtime). In this case the performance achieved is zero.

In this case  $\tau_0 = 5160$  and if  $T_1$  occurs before failure, we clearly see that  $\tau_1$  depends on  $T_1$ .

## 5. Results

Simulation studies allowed us to estimate the proportion of equipment in each state. We simulated 100,000 stories. In Figure 3, we let the system evolve without performing maintenance. Then we obtain the following proportions:

- 18 % of equipment have ball bearing failure,
- 39 % of equipment have electronic failure,
- 43 % of equipment go to degraded state. These equipment have subsequently a failure of the cooling system. In this case the performance equals to zero.

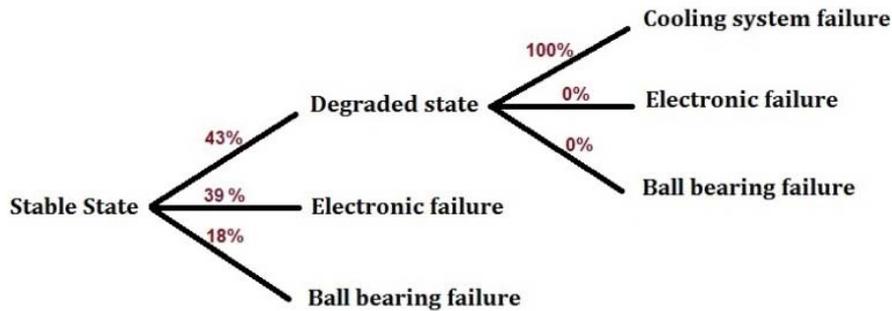


Figure 3: Evolution of the system without maintenance

Now we implement a maintenance policy. The results are given in Figure 4. At time  $t = 0$ , we have the first date of maintenance given by the previous algorithm. At this date, the situation is as follows :

- 30 % of equipments have electronic failure before this date of maintenance,
- 8 % of equipment have ball bearing failure before this date of maintenance,
- 26 % of equipment go to degraded state before this date of maintenance,
- 36 % of equipment are sent for maintenance at the time  $\tau_0$ .

At this moment, a new date is given for maintenance equipment passed in a degraded state. For those gone in this new state 5% fail before this new date of maintenance and 95% are sent for maintenance at this new date. In this case the average performance equals to 2249.

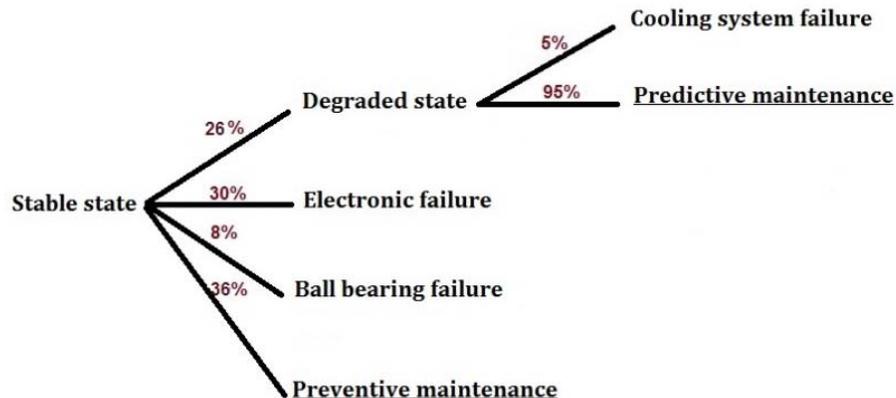


Figure 4: Evolution of the system with maintenance

So using this maintenance policy allows us to recall 61 % of equipment before failure. But if we do not take into account electronic failures, 87 % of equipment are sent for maintenance before failure. Indeed, we cannot perform maintenance on electronic parts, we can only replace it with a new piece. When electronic part of equipment fails, it will not damage the rest of equipment, to repair it is enough to replace as maintenance. So we do not make maintenance on electronic components.

## 6. Conclusion

Estimation of the state of the system associated with a decision criterion should allow to adapt maintenance policies to the observed state of the system, by the detection of failures predictable and a better management of park of equipment available. Thus, Thales will improve its maintenance action, its equipment availability at the lowest cost and the satisfaction of its customers.

## References

- Baysse C., Bihannic D., Gégout-Petit A., Prenat M., Saracco J., 2012, Detection of a degraded operating mode of optronic equipment using Hidden Markov Model, ESREL, Helsinki.
- Cocozza-Thivent C., 1997, Stochastic process and system reliability (Processus stochastique et fiabilité des systèmes), Edition Springer, France.
- Davis M. H. A, 1993, Markov Models and Optimization, Chapman and Hall, London, UK.
- de Saporta B., Dufour F., Zhang H., Elegbede C., 2012, Optimal stopping for the predictive maintenance of a structure subject to corrosion, Journal of Risk and Reliability, 226-(2), 169-181.
- de Saporta B., Dufour F., Gonzalez K., 2010, Numerical method for optimal stopping of piecewise deterministic Markov process, The Annals of Applied Probability, 20(5), 1607-1637.
- Donat R., Leray P., Bouillaut L., Aknin.P, 2010, A dynamic bayesian network to represent discrete duration models, Neurocomputing, 73(4-6), 570-577.
- Lair W., Mercier S., Roussignol M., Ziani.R, 2012, Dynamic modeling of complex systems for reliability calculations and maintenance optimization, Actes du congrès LM 18 - octobre 2012, Tours, France.
- Marsan A., Balbo G., Donatelli S., Franceschinis G., 1995, Modeling with generalized stochastic Petri nets, Wiley, UK.
- Pagès G., Printems J., 2003, Optimal quadratic quantization for numerics: the Gaussian case. Monte Carlo Methods and Applications 9(2), 135-165.