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EBO Optimization Considering the Joint of LORA and Spare Stocks

Jiujiu Fan*, Linhan Guo, Yi Yang, Rui Kang

School of Reliability and System Engineering, Beihang University
37# Xueyuan Road, Haidian District, Beijing 100191, China
mrfan99@163.com

Level of repair analysis (LORA) is an important method of maintenance decision for establishing system of operation and maintenance in the equipment development period. Currently, the researches of equipment of level of repair focus on economic analysis models which used to optimize costs, rarely consider the maintenance time required by the implementation of maintenance program. In fact, maintenance time is an important factor influencing the availability of equipment system. Considering the relationship between maintenance time and spares stocks level, it is obvious that there are contradictions between maintenance time and cost. In order to balance these two influencing factors, it is necessary to build an optimization LORA model. To this end, maintenance time representing performance characteristic is introduced in this article, and on the basis of spares stocks which is traditionally regarded as decision variable, we add a decision variable of repair level, and build a single-echelon, multi-indenture optimization LORA model which takes the best cost-effectiveness ratio as criterion, expected number of backorder (EBO) as objective function and is subject to a constraint on cost. Besides, the paper designs a convex programming algorithm of multi-variable for the optimization model, provides solution of the non-convex objective function and method of improving the efficiency of the algorithm. It is worth mentioning that the optimization model of level of repair designed in the paper is applicable to the engineering.

1. Introduction

So far, there are many scholars make a lot of research on the LORA. Barros (1998) proposes the first multi-echelon, multi-indenture (MEMI) LORA model. Barros and Riley (2001) use the same model as Barros does and solves it using a branch-and-branch approach. Saranga and Dinesh Kumar (2006) make the same assumptions as Barros (1998), except that each component requires its own unique resources. They solve the model by using a genetic algorithm. Basten et al. (2011b) generalize the model by allowing for different decision at various locations at one echelon level, and they show that the LORA problem can be modeled efficiently as a generalized minimum cost flow model.

An amount of literature exists on the spares stocks problem. The paper of Sherbrook (1968) develops the METRIC model, which is the basis for a huge stream of METRIC type models. Muckstadt (1973) extends the work by Sherbrooke (two-echelon, single-indenture) by allowing for two indenture levels, leading to the so-called MOD-METRIC model. Graves (1985) proposes a more accurate approximation for the two-echelon, single-indenture problem, the VARI-METRIC model, which Sherbrooke (1986) extends to two-indenture levels. Axsater (1990) provides an exact evaluation and enumerative, but with penalty costs instead of a service level constraint.

Since the LORA does not consider the availability of the installed base, solving the LORA and spare parts stocking problems sequentially may lead to suboptimal solution. Some researchers solve the problem of LORA and spares stocks jointly. Alfredsson (1997) firstly proposes a two-echelon, single-indenture model jointing LORA and spares stocks. He assumes that each component requires one resource and all components require the same resource is required at the same location. Basten et al. (2011a) propose the same model as Alfredsson (1997), but they allow for more general component-resource and components may share resources.

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However, maintenance time is rarely considered in these traditional researches, it is an important factor that affecting the availability of equipment system. In order to make the maintenance time as soon as possible, we should allocate a certain quantity of spares. Improving the quantity of spares means shortening the maintenance time, but at the same time, costs will be increased. The optimization goal is to achieve the repair level with the best cost-effectiveness ratio and stocks allocation quantity, so the costs may not unlimited growth, and spares stocks level is limited. In order to find the repair level with shortest spares waiting time which is the main time of maintenance time, and lowest costs, we need to synthetically consider the spares waiting time and costs. Using Little formula1, we can transform spares waiting time into EBO, so that we can transform maintenance time-cost balance into EBO-cost balance. In the model, we will take EBO as the objective function, cost as constraint condition, to build an optimization LORA model with considering maintenance time and jointing LORA and spares stocks.

2. Model

2.1 Assumptions and notation

We use the assumptions underlying the MEMI LORA model:

- LRU fail time is exponential distribution;
- There are no lateral transshipments between bases;
- For each component at each location, an (S-1,S) continuous review stocks control policy is used;
- Replacement of a defective component by a functioning component take zero time;
- The repair ability is infinite and repairs are always successful;
- Except for spare parts, other maintenance resources are always adequate;
- A failure in a LRU is caused by a failure in at most one SRU

We define the following notations:

m_{i0}^{Oa}: LRU$_i$ demand ratio at organization-level *Oa*

m_{i0}^{Ib}: LRU$_i$ demand ratio at intermediate-level *Ib*

m_{i0}^{Dc}: LRU$_i$ demand ratio at depot-level *Dc*

p_{i1}^{Oa} : Probability of organization-level's failure LRU$_i$ repair at organization-level

p_{i2}^{Oa} : Probability of organization-level's failure LRU$_i$ deliver to intermediate-level to repair

p_{i3}^{Oa} : Probability of organization-level's failure LRU$_i$ deliver to depot-level to repair

p_{i2}^{Ib}: Probability of intermediate-level's failure LRU$_i$ repair at intermediate-level

p_{i3}^{Ib}: Probability of intermediate-level's failure LRU$_i$ deliver to depot-level to repair

T_{i0}^{Oa}: Repair time of LRU$_i$ at organization-level

T_{i0}^{Ib}: Repair time of LRU$_i$ at intermediate-level

T_{i0}^{Dc}: Repair time of LRU$_i$ at depot-level

T_{wi0}^{Ib}: LRU$_i$ waiting supply time at intermediate-level

T_{wi0}^{Dc}: LRU$_i$ waiting supply time at depot-level

2.2 Mathematical model

In the paper, we mainly analyze single-echelon, three-indenture support system and build an optimization model. As shown in Figure 1.

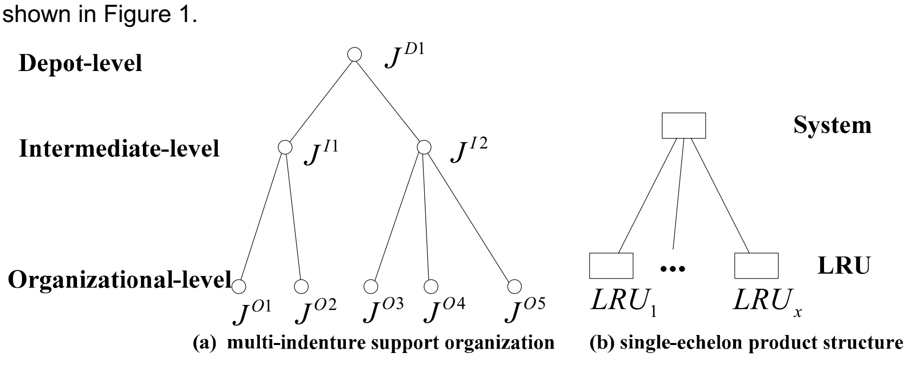

Figure 1 single-echelon, three-indenture support structure

In section 1, we have analyzed that the objective function of model is EBO which is transformed by spares waiting time. Therefore, the real objective function is spares waiting time. Now, we deduce the relationship between spares waiting time and EBO.

$$T_w = \frac{\sum_{a=1}^{A} m^{Oa} T_w^{Oa}}{\sum_{a=1}^{A} m^{Oa}} \rightarrow T_w^{Oa} = \frac{\sum_{i=1}^{I(a)} m_i^{Oa} T_{wi}^{Oa}}{\sum_{i=1}^{I(a)} m_i^{Oa}} \rightarrow T_{wi}^{Oa} = \frac{EBO_i^{Oa}}{m_i^{Oa}}$$

$$\Downarrow$$

$$T_w = \frac{\sum_{a=1}^{A} \sum_{i=1}^{I(a)} EBO_i^{Oa}}{\sum_{a=1}^{A} \sum_{i=1}^{I(a)} m_i^{Oa}}$$

Figure 2 The relationship of EBO and mean spares waiting time

As shown in Figure 2, when the demand of each site is known, that achieving minimum possible mean waiting time is equal to achieving minimum possible $\sum_{a=1}^{A} \sum_{i=1}^{I(a)} EBO_i^{Oa}$, therefore, the objective function is $\sum_{a=1}^{A} \sum_{i=1}^{I(a)} EBO_i^{Oa}$. We analyze the objective function as follows:

$$\sum_{a=1}^{A} \sum_{i=1}^{I(a)} EBO_i^{Oa} \rightarrow EBO(s_{i0}^{Oa} \mid E[X_{i0}^{Oa}])$$

$$E[X_{i0}^{Oa}] = m_{i00}^{Oa} \{T_{i0}^{Oa} p_{i1}^{Oa} + (O_{i0}^{IbOa} + T_{wi0}^{Ib}) p_{i2}^{Oa} + (O_{i0}^{DcOa} + T_{wi0}^{Dc}) p_{i0}^{Oa}\}$$

$$T_{wi0}^{Ib} = EBO(s_{i0}^{Ib} \mid E[X_{i0}^{Ib}]) / m_{i0}^{Ib} \qquad T_{wi0}^{Dc} = EBO(s_{i0}^{Dc} \mid E[X_{i0}^{Dc}]) / m_{i0}^{Dc}$$

$$E[X_{i0}^{Ib}] = \sum_{Oa \subset Ib} m_{i00}^{Oa} p_{i2}^{Oa} \{T_{i0}^{Ib} p_{i2}^{Ib} + (O_{i0}^{DcIb} + T_{wi0}^{Dc}) p_{i3}^{Ib}\}$$

$$E[X_{i0}^{Dc}] = m_{i0}^{Dc} T_{i0}^{Dc}$$

Figure 3 Computing flow for MEMI objective function

In Figure 3, we present how to compute the objective function EBO. Transforming spares waiting time into EBO is the key. By dissecting the EBO expression, we can compute the number of LRU and SRU in repair at each indenture site to get the value of spares waiting time and finally get the EBO.

Above all, we can get mathematic model. We define EBO as objective function, and we can get $EBO(s_{i0}^{Oa} \mid E[X_{i0}^{Oa}])$ according to calculating flow as show in *figure 3*. By analyzing in section 2.1, we need to consider two constrains condition: costs and repair level decision variable which is 0-1 variable. Mathematic model is shown in (1).

$$\min \sum_{a=1}^{A} \sum_{i=1}^{I(a)} EBO(s_{i0}^{Oa} \mid E[X_{i0}^{Oa}])$$

$$s.t. \begin{cases} c_{i0}^{s}(\sum_{a=1}^{A} s_{i0}^{Oa} + \sum_{b=1}^{B} s_{i0}^{Ib} + \sum_{c=1}^{C} s_{i0}^{Dc}) \le C_m \\ p_{i1}^{Oa}, p_{i2}^{Oa}, p_{i3}^{Oa}, p_{i2}^{Ib}, p_{i3}^{Ib} \in \{0,1\}, i = 1,2,...,I \\ p_{i1}^{Oa} + p_{i2}^{Oa} + p_{i3}^{Oa} = 1, i = 1,2,...,I \\ p_{i2}^{Ib} + p_{i3}^{Ib} = 1, \text{if } p_{i2}^{Oa} = 1, i = 1,2,...,I \\ p_{i2}^{Ib} + p_{i3}^{Ib} = 0, \text{if } p_{i2}^{Oa} = 0, i = 1,2,...,I \end{cases} \quad (1)$$

3. Algorithm

Differ from objective function EBO(s) in traditional METRIC type model, we add a repair level decision variable p in our model, so that stocks is not the only variable in objective function EBO(s,p) which contain stocks s and repair level p two variables. Although EBO presents convex function characteristic in stocks dimension, it is not explicit in repair level dimension. It is easy to prove EBO(s,p) is no-convex, so we should improve algorithm to make EBO(s,p) convex.

While traverse the repair level combination of all kinds of spares, the circulation of repair level combination inserted in the circulation of spares type. Therefore, if we have structured the EBO convex curve of all kinds of spares according to repair level decision variable before traversing the spares types, all kinds of spares can be made convex optimization according to the margin iteration of unit cost effect. We can come to a conclusion that the key step of solving non-convex function optimization algorithm is to structure convex function respectively for non-convex function EBO(p) of all kinds of spares.

The method of structuring convex function EBO(p)-cost of all kinds of spares is the same as structuring EBO(s)-cost optimization curve by increasing the unit cost effect of the certain spares to make optimization decision. To a certain spare, there may be several repair level to choose, we fix the stocks s of EBO(s,p), compare which kind of choice to make EBO(p) minimum, then choose the minimum EBO(p) of repair level in each step of increasing spares stocking, and we can get the optimization EBO(p)-cost convex function.

We structure EBO(p)-cost curve according to the theory of marginal analysis, so it is obvious that the curve is convex. Using the point on EBO(p)-cost convex curve to make optimization analysis among all kinds of spares, we can get the optimization curve of EBO(s,p)-cost. The whole algorithm flow is shown in Figure 4.

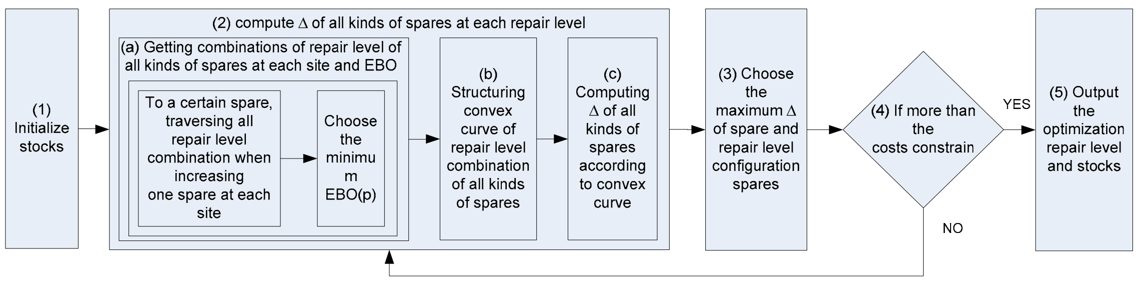

Figure 4 Algorithm flow diagram

4. Application case

Taking equipment maintenance planning as background, we introduce how to apply the convex optimization algorithm to level of repair analysis, modeling, solve and proving in support system according to optimization theory proposed in this paper.

In this case, we build a three-indenture, single-echelon optimization model as shown in Figure 1. We use simple variable method to analyze the impact of maintenance time, unit cost and demand rate. As shown in Table 1, LRU_1 and LRU_2 are only different from maintenance time, LRU2 and LRU_3 are only different from unit cost, and LRU_3 and LRU_4 are only different from demand ratio.

For multi-type LRU, we apply the algorithm designed in this paper to compute and in return prove the feasibility of algorithm.

(1) Input date

Table 1: Input date of multi-site, multi-type LRU

LRU	Demand Ratio	Dc repair time (year)	Ib repair time (year)	Oa repair time (year)	DI Transportation time(year)	DO Transportation time(year)	IO Transportation time(year)	Unit cost (thousand dollar)	Costs constrain (thousand dollar)
LRU_1	12.438	0.02	0.0149	0.0418	0.0359	0.0150	0.0570	4	85
	16.691	0.02	0.0149	0.0360	0.0359	0.0146	0.0839	4	
	15.215	0.02	0.0235	0.0384	0.0217	0.0307	0.0658	4	
	9.411	0.02	0.0235	0.0577	0.0217	0.0123	0.0897	4	
LRU_2	12.438	0.02	0.0132	0.0203	0.0392	0.0345	0.0742	4	
	16.691	0.02	0.0132	0.0327	0.0392	0.0144	0.0385	4	
	15.215	0.02	0.0229	0.0259	0.0382	0.0178	0.0480	4	
	9.411	0.02	0.0229	0.0809	0.0382	0.0265	0.0606	4	
LRU_3	12.438	0.02	0.0132	0.0203	0.0392	0.0345	0.0742	5	
	16.691	0.02	0.0132	0.0327	0.0392	0.0144	0.0385	5	
	15.215	0.02	0.0229	0.0259	0.0382	0.0178	0.0480	5	
	9.411	0.02	0.0229	0.0809	0.0382	0.0265	0.0606	5	
LRU_4	3.508	0.02	0.0132	0.0203	0.0392	0.0345	0.0742	5	
	4.211	0.02	0.0132	0.0327	0.0392	0.0144	0.0385	5	
	2.136	0.02	0.0229	0.0259	0.0382	0.0178	0.0480	5	
	9.568	0.02	0.0229	0.0809	0.0382	0.0265	0.0606	5	
LRU_5	4.487	0.02	0.0264	0.0721	0.0259	0.0306	0.0332	8	
	6.821	0.02	0.0264	0.0541	0.0259	0.0234	0.0505	8	
	2.541	0.02	0.0379	0.0458	0.0355	0.0334	0.0638	8	
	16.102	0.02	0.0379	0.0556	0.0355	0.0345	0.0558	8	

(2) Output

Costs constrain is 85, and the optimization value of EBO is 1.597809235310

Repair Combinations:

Intermediate-level (1) site: Organization-level (1) site: failure LRU_1 sends to Depot-level site to repair;
Intermediate-level (1) site: Organization-level (2) site: failure LRU_1 sends to local site to repair;
Intermediate-level (1) site: Organization-level (1) site: failure LRU_2 sends to local site to repair;
Intermediate-level (1) site: Organization-level (2) site: failure LRU_2 sends to local site to repair;
Intermediate-level (1) site: Organization-level (1) site: failure LRU_3 sends to local site to repair;
Intermediate-level (1) site: Organization-level (2) site: failure LRU_3 sends to local site to repair;
Intermediate-level (1) site: Organization-level (1) site: failure LRU_4 sends to local site to repair;
Intermediate-level (1) site: Organization-level (2) site: failure LRU_4 sends to local site to repair;
Intermediate-level (1) site: Organization-level (1) site: failure LRU_5 sends to Depot-level site to repair;
Intermediate-level (1) site: Organization-level (2) site: failure LRU_5 sends to Depot-level site to repair;
Intermediate-level (2) site: Organization-level (1) site: failure LRU_1 sends to Depot-level site to repair;
Intermediate-level (2) site: Organization-level (2) site: failure LRU_1 sends to Depot-level site to repair;
Intermediate-level (2) site: Organization-level (1) site: failure LRU_2 sends to local site to repair;
Intermediate-level (2) site: Organization-level (2) site: failure LRU_2 sends to Depot-level site to repair;
Intermediate-level (2) site: Organization-level (1) site: failure LRU_3 sends to local site to repair;
Intermediate-level (2) site: Organization-level (2) site: failure LRU_3 sends to Depot-level site to repair;
Intermediate-level (2) site: Organization-level (1) site: failure LRU_4 sends to local site to repair;
Intermediate-level (2) site: Organization-level (2) site: failure LRU_4 sends to Depot-level site to repair;
Intermediate-level (2) site: Organization-level (1) site: failure LRU_5 sends to Depot-level site to repair;
Intermediate-level (2) site: Organization-level (2) site: failure LRU_5 sends to Depot-level site to repair.

Table 2: stocks allocation of multi-site, multi-type LRU

LRU_i	Depot-level	Intermediate-level		Organization-level			
		B1	B2	A1(b1)	A2(b1)	A1(b2)	A2(b2)
LRU_1	1	0	0	1	1	2	1
LRU_2	0	0	0	1	2	1	1
LRU_3	0	0	0	1	1	1	1
LRU_4	0	0	0	0	0	0	1
LRU_5	1	0	0	0	0	0	1

As shown in Figure 5, EBO_i-C curve and EBO-C curve.

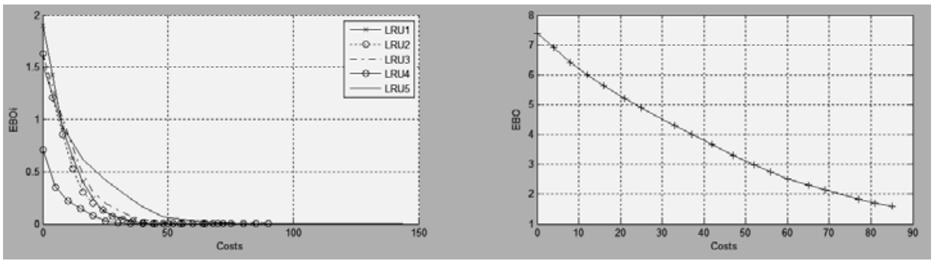

Figure 5 Optimal curve of multi-type LRU

(3) Result analysis

From the result, when compared with LRU_1 to LRU_5, we can get that maintenance time of LRU at each site can affect the combination of repair level, demand rate and unit cost of LRU can affect stocking allocation at each site. Model, algorithm and program provided in paper can solve the optimization problem of any number of sites and LRU in three-indenture support organization.

5. Conclusions and further research

In this paper, we propose a method of LORA considering maintenance time in equipment support system.
(1)Build single-echelon, multi-indenture optimization model jointing LORA and spares stocks
Deduce the LORA objective function and give the modeling condition, provide the optimal objective function formula and constraint condition at support organization, build optimization model.
(2)Design convex optimization algorithm for model
Analyze the characteristics of optimization problem, design multi-variable convex optimization algorithm, expound the theory and flow of algorithm, and provide solution of non-convex objective function and method of improving the efficiency of algorithm.
Further research:
(1)Consider all maintenance resources, build optimization model of limited maintenance capability;
(2)Take the correlation of inter-depot maintenance resources into account.

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