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Reliability Estimate of Probabilistic-Physics-of-Failure Degradation Models

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It is universally accepted that physics-based models provide more accurate estimates of reliability than statistics-based model especially when the physics of failure of the units are well-understood. However, the underlying assumptions of such models regarding the deterministic nature of its parameters limit their applications, implementations and generalization. In this paper, we propose a physic-based accelerated degradation model that considers probability distributions of its parameters. The degradation path is then determined, and the first passage distribution, failure time function and its parameters' estimation are obtained accordingly. The model is then used to estimate competing risks at normal operating conditions. A numerical example is provided to demonstrate the use of the model and validate its estimates.

1. Introduction

Researches have presented a wide variety of models for predicting the life span of materials and components using accelerated degradation testing (ADT). In general, there are three different kinds of degradation models: physics-statistics-based models, parametric statistics-based models and nonparametric statistics-models. The physics-statistics-based models have a merit of understanding the degradation mechanism of material and components which result in more accurate predictions of the reliability metrics. Physics-of-Failure (PoF) are proposed by Rome Air Development Center (RADC) of the USA Air Force to address the growing complexity of military equipment, and the consequent higher numbers of observed failures. Thereafter the PoF have been extensively studied in reliability area and achieved significant successes (Ebel, 1998). Alpern and Lee, (2008) identify the moisture concentration at the weakest material interface as the critical parameter for the defect onset, and predict the life of moisture-induced delamination between the modeling compound and die surface in plastic packages using an ordinary 1-D diffusion model. Jin et al. (2012) propose a novel reliability estimation framework in terms of physics of failure based on the relationship between the physics performance and its failure mechanisms. Based on the failure model, Jin et al. (2013) develop ta life prediction method for a momentum wheel in a dynamic covariate environment. Reliability Information Analysis Centre (RIAC) presents an approach for microelectronic system reliability assessment and qualification based on PoF in a sum-of-rate to account for multiple mechanisms (Salemi et al., 2008).

Due to uncertainties and the stochastic nature of failures, the PoF cannot be generalized or effectively applied in many situations. So the Probabilistic Physics of Failure (PPoF) have been introduced in the reliability field (Mendel, 1996). There are three main sources of variations: (1) environmental factors that arise from field environment or testing conditions; (2) the unknown and dynamical value of practical mission profiles; (3) the variation caused by manufacturing processes (Matic and Sruk, 2008). Hall and Strutt (2003) present PPoF for both Eyring model and Power-Law model considering environmental parameters (e.g. temperature, humidity). Chatterjee and Modarres (2012) present a PPoF approach for estimating tube rupture frequency in steam generator of water reactor, which regards PoF models as a probabilistic form associated with environmental conditions, geometrical and material properties. Haggag et al. (2000) use an PPoF approach to predict the reliability of high-performance chips that considers the activation energies follow a finite binomial distribution. Even though there are several PPoF models for life

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time prediction at different operating conditions, there are limited (or non-existent) degradation functions of PoF approaches which consider both uncertainties of material properties and their activation energies. Extensive research shows that the activation energy can be described by a probabilistic distribution but most of the PPoF treat it as a deterministic characteristic of the material.

In this paper, we propose a degradation function of PPoF which considers the probability distributions of materials characteristics. Thereafter, this function is modeled as a Brownian process and its parameters are estimated accordingly. We then develop a competing risk PPoF degradation model, when units exhibits several failure modes, and obtain the corresponding reliability function. A simulation example is demonstrated for model validation.

2. Probabilistic physics of failure degradation models

PoF degradation models are usually developed for specific applications and conditions. For example, there are several PoF models for electromigration of semiconductors which describe the Time Dependent Dielectric Break Drown (TDDB), Hot Carrier Injection (HCI) and Negative Bias Temperature Instability (NBTI). More generalized performance degradation models of PoF include the Arrhenius dependence, Power dependence, exponential dependence and Eyring dependence models. In this paper, we consider the Arrhenius relationship given by Eq. (1).

$$D(t) = D(0) - t\beta \exp(-E/T) \tag{1}$$

$D(t) =$ degradation value at time t, $D(0) =$ initial degradation value, $\beta =$ characteristic of the product or properties of material, $E =$ activation energy, $T =$ operating temperature in Kelvin.

We consider two sources of variations: the first is due to unit to unit product characteristics β and the activation energy variation E. Therefore, it is reasonable to assume that β and E to be distributed by two Normal distributions, say $E \sim N(\mu_1, \sigma_1^2)$ and $\beta \sim N(\mu_2, \sigma_2^2)$ as described in (Braun and Burnham, 1987, Anthony and Howard, 2004).

Taking the logarithm of Eq. (1) and setting $D(0) = 0$ we obtain

$$\ln D(t) = \frac{E}{T} - \ln t - \ln \beta \tag{2}$$

Expanding $\ln \beta$ using Taylor series about $\beta = \mu_2$, Eq. (2) becomes

$$Y(\ln t) = \ln D(t) \approx \frac{E}{T} - \ln t - \frac{\beta}{\mu_2} = W - \ln t \tag{3}$$

It is known that the sum of two normal distributions also follows a normal distribution, so the distribution of W is:

$$W \sim N(\mu_1 / T - \mu_2 / \sigma_2, 1 + \sigma_1^2 / T^2) \tag{4}$$

3. Stochastic process model of PPoF

Let $L(t)$ denotes the degradation path of a specific unit. We also assume that $L(t)$ is a decreasing function which has a failure threshold value L_γ therefore, the lifetime could be obtained by the First Passage Time (FPT), which is,

$$Z = \inf\{t \mid L(t) \ll \omega\} \tag{5}$$

As stated earlier, there are uncertainties in the degradation data which can be included in PoF model:

$$L(t) = Y(t) + \varepsilon(t) \tag{6}$$

where $Y(t)$ is the actual degradation value, $\varepsilon(t) = \lambda B(t)$ is the uncertainty in the degradation measurements, $B(t)$ is a standard Brownian motion. We rewrite Eq. (6) as

$$L(\ln t) = Y(\ln t) + \varepsilon(\ln t) = W - \ln t + \lambda B(\ln t) \tag{7}$$

where $W \sim N(\mu_1 / T - \mu_2 / \sigma_2, 1 + \sigma_1^2 / T^2)$.

According to Basak and Balakrishnan (2011), Eq. (7) is a Brownian motion process with initial value w, drift λ and diffusion $-\ln t$. It is known that for a monotone degradation path the First Passage Time (FPT) distribution follows an Inverse Gaussian (IG) distribution. In our case the FPT distribution of Eq. (7) and its unreliability function are obtained as,

$$f(\ln t; L_\gamma W, -1, \lambda) = \frac{\sqrt{\lambda / 2\pi(\ln t - L_\gamma - W)^3}\,\exp(-\lambda(\ln t - L_\gamma - W + 1)^2)}{2(\ln t - L_\gamma - W)}, (\ln t > 1) \tag{8}$$

and

$$F(\ln t; L_\gamma W, -1, \lambda) = \Phi\left[\sqrt{\frac{\lambda}{\ln t - L_\gamma - W}}(\ln t - L_\gamma - W - 1)\right] + e^{-2\lambda}\Phi\left[-\sqrt{\frac{\lambda}{\ln t - L_\gamma - W}}(\ln t - L_\gamma - W + 1)\right] \tag{9}$$

where $\Phi(\tau) = \int_{-\infty}^{\tau} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$, w is the initial value, L_γ is the threshold value, λ is the scale parameter, and $\mu + \lambda$ is the mean.

4. Parameters' estimation

Consider a failure mode k which has a degradation value $L_{k,j,i}$ for path j at time t_i ($k = 1 \cdots Q, j = 1 \cdots M, i = 1 \cdots N$). According to the properties of Brownian motion, the increments are *i.i.d.* distributed by a normal distribution, therefore the *p.d.f.* of increments for degradation path *j* of unit *k* is given by (Tseng and Peng, 2007),

$$f_{k,j,i}(\Delta L_{k,j,i}) = \frac{1}{\lambda_k \sqrt{\Delta \ln t_i}} \phi\left[\frac{\Delta L_{k,j,i} + \Delta \ln t_i}{\lambda_k \sqrt{\Delta \ln t_i}}\right] \tag{10}$$

where

$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, $\Delta_i L_{k,j} = L_{k,j,i+1} - L_{k,j,i}$, $\Delta \ln t_i = \ln t_{i+1} - \ln t_i$ and λ_k is the drift for path *j* of failure mode k.

In order to estimate the parameters of Brownian motion process, we apply MLE method to Eq. (10) for all the paths of failure k. The log-likelihood function for failure mode *k* is

$$\ln L_k = -MN \ln \lambda_k - \sum_{j=1}^{M}\sum_{i=1}^{N} \ln \sqrt{\Delta \ln t_i} + \sum_{j=1}^{M}\sum_{i=1}^{N} \ln \phi\left[(\Delta L_{k,j,i} + \Delta \ln t_i) / \lambda_k \sqrt{\Delta \ln t_i}\right] \tag{11}$$

The unknown parameter $\hat{\lambda}_k$ is obtained by solving $dL_k / d\lambda_k = 0$, and obtain

$$\hat{\lambda}_k = \left\{ \frac{3\sum_{j=1}^{M}\sum_{i=1}^{N}\left[\frac{1}{N}(\Delta L_{k,j,i} + \Delta \ln t_i)^2 / \Delta \ln t_i\right]}{\Theta(3\sum_{j=1}^{M}\sum_{i=1}^{N}\left[\frac{1}{N}(\Delta L_{k,j,i} + \Delta \ln t_i)^2 / \Delta \ln t_i\right])} \right\}^{1/3} \tag{12}$$

where $\Theta(x)$ is the root of $y = xe^x, (-1 \le y < \infty), \theta = -1, 3, -3$.

5. PPoF Competing risk model

We consider a system with k failure modes represented by $k(k = 1 ... Q)$, and develop its Competing Risk Model (CRM). Based on the aforementioned assumptions, under the competing risk model, the system fails when any of the failure modes reaches the corresponding threshold first. Therefore, the failure function of the system is the product of each resulting failure function. We further assume that the modes are due to the significant variations of activation energies and material properties. That is, units having the same failure mode are deemed to have the same $\hat{W}_k$. Based on Eq. (9), the failure function of failure mode *k* is given by,

$$F_k = \Phi\left[\sqrt{\frac{\lambda}{\ln t - L_\gamma - W_k}}(\ln t - L_\gamma - W_k - 1)\right] + e^{-2\lambda}\Phi\left[-\sqrt{\frac{\lambda}{\ln t - L_\gamma - W_k}}(\ln t - L_\gamma - W_k + 1)\right] \tag{13}$$

We obtain the CRM of the failure function (Elsayed, 2012) as

$$
\begin{aligned}
F(\ln t) &= 1-\prod_{k=1}^{Q}[1-F_k(\ln t)] \\
&= 1-\prod_{k=1}^{Q}\left\{1-\Phi\left[\sqrt{\frac{\lambda}{\ln t-L_\gamma-W_k}}(\ln t-L_\gamma-W_k-1)\right]-e^{-2\lambda}\Phi\left[-\sqrt{\frac{\lambda}{\ln t-L_\gamma-W_k}}(\ln t-L_\gamma-W_k+1)\right]\right\}
\end{aligned}
\tag{14}
$$

6. Mote Carlo simulation of PPoF competing risk model

We consider a unit that exhibits ten different failure modes and generate 10 paths for each mode. The threshold is set to 1 for all the units. Each path is given by a Brownian motion process with diffusion $-\ln t$, drift 2 and initial value $W_k \sim N(2,1)$ as shown in Figure 1 and the simulation flow chart is shown in Figure 2

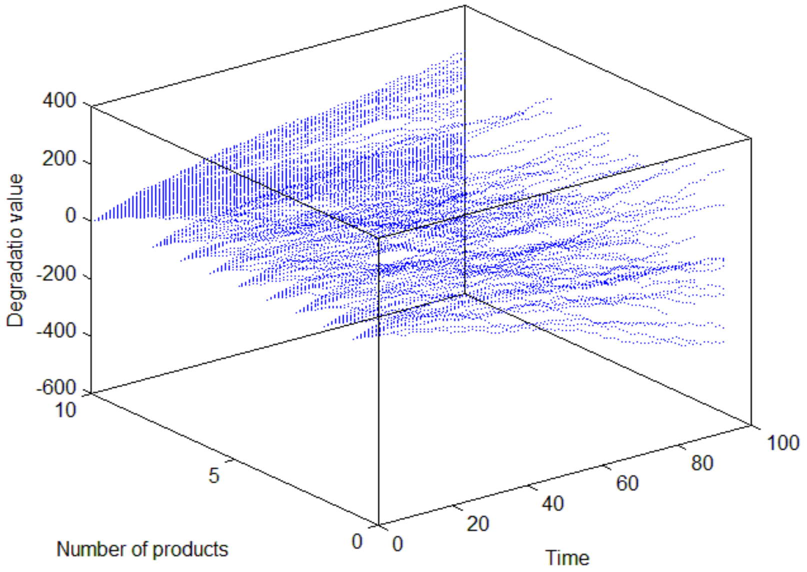

Figure 1 Generating degradation paths for all the units

We assume that these failure modes occur due to the variation of parameter W_k, which has a known distribution. We use Eq. (12) to estimate the drift $\hat{\lambda}_k$ for every failure mode. The failure functions can be obtained by Eq. (13) accordingly and the CRM function is given by Eq. (14).

Finally, the failure function of the CRM is obtained as

$$
\begin{aligned}
F(\ln t) = 1- & \\
& \left\{1-\Phi\left[\sqrt{\frac{0.74}{\ln t-2.67}}(\ln t-0.38)\right]-e^{-1.48}\Phi\left[-\sqrt{\frac{0.74}{\ln t-2.67}}(\ln t-0.38)\right]\right\}_1 \ldots \\
& \times\left\{1-\Phi\left[\sqrt{\frac{1.26}{\ln t-2.36}}(\ln t-1.61)\right]-e^{-2.52}\Phi\left[-\sqrt{\frac{1.26}{\ln t-2.36}}(\ln t-1.61)\right]\right\}_{10}
\end{aligned}
\tag{15}
$$

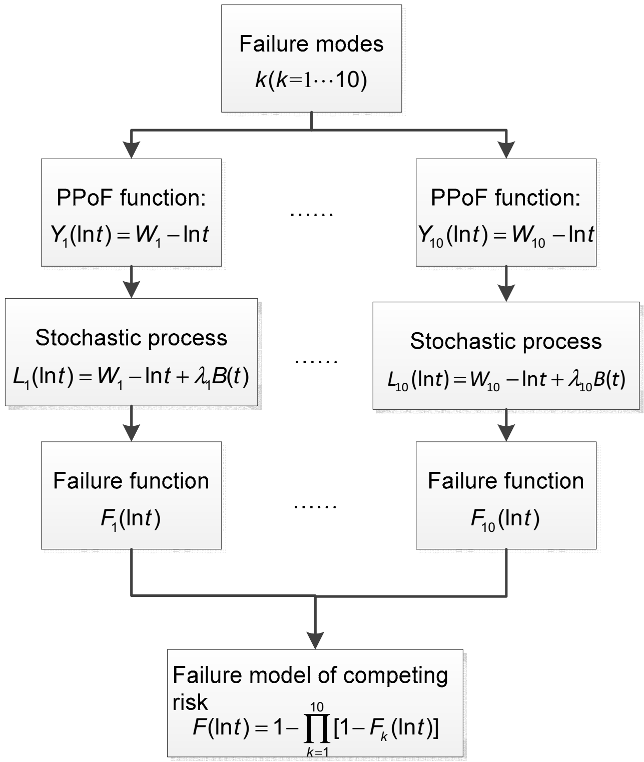

Figure 2. Flow chart of the process used in the competing risk model of PPoF

Table 1: Results of estimation

Parameter	1	2	3	4	5	6	7	8	9	10
$\hat{\lambda}_k$	0.74	1.54	0.9	1.41	0.59	0.6	0.99	0.74	0.74	1.26
$\lvert\lambda-\hat{\lambda}\rvert/\lambda$	0.12	0.08	0.01	0.12	0.02	0.22	0.21	0.21	0.01	0.12
W_k	2.67	1.33	1.6	1.33	2.58	1.22	0.94	2.55	1.58	2.36

7. Summary

PPoF model has its merits for predicting reliability based on degradation observations. In this paper, the variation of individual units, the activation energy and uncertainty in degradation measurements are considered in the traditional Arrhenius degradation function. The corresponding FPT distribution and its failure function are developed. We also develop a failure function of the CRM for different failure modes and present a simulation example to demonstrate the application of the proposed model.

References

Alpern P., Lee K.C, 2008, Moisture-induced delamination in plastic encapsulated microelectronic devices: a physics of failure approach. IEEE Transactions on Device and Materials Reliability, 8, 478-483.

Anthony D.B., Howard J.B., 2004, Coal devolatilization and hydrogastification. AICHE Journal, 22, 625-656.

Basak P., Balakrishnan N, 2011, Estimation for the three-parameter inverse Gaussian distribution under progressive type-II censoring. Journal of Statistical Computation and Simulation, 82, 1055-1072.

Braun R., Burnham A., 1987, Analysis of chemical reaction kinetics using a distribution of activation energies and simpler models. Energy & Fuels, 1, 153-161.

Chatterjee K., Modarres M., 2012, A probabilistic physics-of-failure approach to prediction of steam generator tube rupture frequency. Nuclear Science and Engineering, 170, 136-150.

Ebel G.H., 1998, Reliability physics in electronics: a historical view. IEEE Transactions on Reliability, 47, 379-389.

Elsayed E.A., 2012, Reliability Engineering. Wiley, New York, USA.

Haggag A., McMahon W., Hess K., Cheng K., Lee J., Lyding J., 2000, A probabilistic-physics-of-failure/short-time-test approach to reliability assurance for high-performance chips: models for deep-submicron transistors and optical interconnects. Integrated Reliability Workshop Final Report, IEEE International, 2000, IEEE, 179-182.

Hall P.L., Strutt J.E., 2003, Probabilistic physics-of-failure models for component reliabilities using Monte Carlo simulation and Weibull analysis: a parametric study. Reliability Engineering and System Safety, 80, 233-242.

Jin G., Liu Q., Zhou J.L., Zhou Z.B., 2012, RePofe: Reliability physics of failure estimation based on stochastic performance degradation for the momentum wheel. Engineering Failure Analysis, 22, 50-63.

Jin G., Matthews D., Fan Y., Liu Q., 2013, Physics of failure-based degradation modelling and lifetime prediction of the momentum wheel in a dynamic covariate environment. Engineering Failure Analysis, 28, 222-240.

Matic Z., Sruk V., 2008, The Physics-of-Failure approach in reliability engineering. Information Technology Interfaces, 30th International Conference, 2008, IEEE, 745-750.

Mendel M., 1996, The case for probabilistic physics of failure. NATO ASI Series F Computer and Systems Sciences, 154, 70-82.

Salemi S., Yang L., Dai J., Qin J. Bernstein J.B., 2008, Physics-of-Failure Based Handbook of Microelectronic Systems, RIAC, Maryland, USA.

Tseng S.T., Peng C.Y, 2007, Stochastic diffusion modelling of degradation data. Journal of Data Science, 5, 315-333.