

# Accelerated Degradation Test and Particle Filter Based Remaining Useful Life Prediction

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This paper presents a particle filtering based long-term RUL prediction method that integrates two data resources: infield Accelerated Degradation testing (ADT) and field operation. This method improves the usage of historical information and makes accurate residual life prediction compared with conventional regression method. ADT data is used as prior information to establish dynamic system model by stochastic degradation process modelling, and more specifically drift Brownian motion. Then particle filtering is introduced to estimate system state or forecast residual life. Two stages are included which are on-line filtering and off-line prediction. The proposed method is validated through experimental data and operational data of Super Luminescent Diode.

## 1. Introduction

Rapid development of scientific technology promotes the maturity of product design and manufacturing process. High-reliability and long-life product has been widely used in many fields during past decades. State monitor and reliability evaluation for such product in service phase is definitely difficult. At first, monitor at less times or wrong-time will easily ignore failures which may lead to catastrophic events happening and result in heavy property loss, casualties or injuries. Meanwhile, frequent state monitor cause the waste of human and financial resource. Hence, remaining useful life (RUL) prediction is a key issue in prognostics and health management (PHM) (Si et al., 2011).

Nowadays, many RUL prediction methods are introduced. Algorithms like conventional regression method, Neural Network (Zio et al., 2012), Support Vectors Machine, etc., provide available tools for RUL prediction and reliability estimate (Sikorska et al., 2011).

When using aforementioned methods for high-reliability and long-life products, RUL prediction needs to take cost and time constraint into consideration. In order to obtain accurate results, more time should be needed to collect sufficient data. This limits the development of RUL prediction in time and financial aspects. An appropriate choice to overcome such obstacle in engineering is the implement of Accelerated Degradation Testing (ADT). For ADT, by using more severe testing conditions to accelerate performance degradation process than that experienced in normal condition, more performance information would be collected in a shorter time. Researching on ADT mainly focus on degradation models (Escobar and Meeker, 2006) and design (Ge et al., 2011). Step Stress ADT (SSADT) is preferred in practical because of the advantages of fewer test samples, shorter test time and more stress levels. (Tang et al., 2004).

Recently, particle filtering (PF) based life prediction model has been widely used in RUL prediction since it is suitable for describing any stochastic process represented by dynamic system model (Chen et al., 2012). State transfer process can be expressed using state equation and measurement equation. And system uncertainty can be well captured without any constraint to state and measurement noises. PF breaks through the assumption of linearity/Gaussian for Kalman filtering and therefore extends the RUL prediction area (Zio and Peloni, 2011). For specific details of particle filtering, see (Arulampalam et al., 2002).

In this paper, a long-term RUL prediction method is proposed based on the theory of ADT and PF. With the comprehensive using of laboratory test and field operation, the proposed method can improve RUL

prediction dependability and accuracy under the constraint of time and cost for high-reliability and long-life product.

## 2. Theoretical background

### 2.1 Accelerated degradation testing

To describe degradation progress, drift Brownian motion (DBM), i.e. Wiener process, is introduced since stochastic progress is widely used for this purpose. Degradation model is given by (Li et al., 2010)

$$Y(t) = \sigma B(t) + d(s) \cdot t + y_0 \quad (1)$$

where  $Y(t)$  is performance degradation value,  $\sigma$  is diffusion coefficient which is assumed to be constant,  $B(t)$  is a standard Brownian motion following  $N(0, t)$ ,  $y_0$  is the initial degradation value,  $d(s)$  is the drift coefficient which is only related to stress  $s$  and described as degradation rate in ADT, i.e.,

$$d(s) = \exp(A + B\phi(s)) \quad (2)$$

As the property of DBM, the increment of performance degradation  $\Delta y$  follows normal distribution, i.e.,  $\Delta y \sim N(d(s)\Delta t, \sigma^2 \Delta t)$ . Rewrite it as

$$y(t+1) - y(t) = d(s) \Delta t + n_{\Delta t} \quad (3)$$

where  $n \sim N(0, \sigma^2 \Delta t)$ . Equation 2 and 3 establish the relationship between degradation measurements and stress level which can be considered as dynamic system model. Model parameters mentioned above can be estimated based on ADT data by using least square and maximum likelihood estimation (Ge et al., 2011). Then RUL prediction can be made giving failure level  $l$ .

### 2.2 Particle filtering

Particle filtering is a Bayesian Monte Carlo method that employs a sequential importance sampling algorithm which offers a probabilistic solution framework of state estimate for linear and non-linear systems without any assumption of linearity and Gaussian. A generic dynamic system model is given by

$$x_i = f(x_{i-1}, v_{i-1}) \quad (4)$$

$$y_i = h(x_i, n_i) \quad (5)$$

where  $v_i$  and  $n_i$  denote state and measurement noise. State equation and measurement equation of dynamic system model are represented by Equation 4 and 5. Figure 1 shows the process of PF inference.

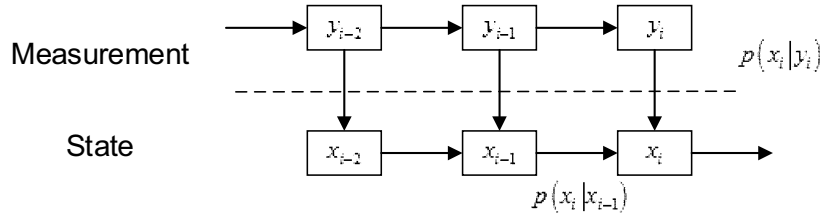


Figure 1: A brief process of particle filtering inference

Here we assume that state transfer process follows first-order Markov process and observations are mutually independent. In Bayesian perspective, state estimate consists of two stages: prediction and update (Arulampalam et al., 2002). Suppose that probability density function (pdf) at time  $i-1$  is known, i.e.  $p(x_{i-1} | y_{1:i-1})$ . Combining Equation 4 with Chapman-Kolmogorov equation, prediction step goes as

$$p(x_i | y_{1:i-1}) = \int p(x_i | x_{i-1}) p(x_{i-1} | y_{1:i-1}) dx_{i-1} \quad (6)$$

When new measurement at time  $i$  is available, update step goes as

$$p(x_i | y_{1:i}) = p(y_i | x_i) p(x_i | y_{1:i-1}) / p(y_i | y_{1:i-1}) \quad (7)$$

where  $p(y_i | y_{1:i-1}) = \int p(y_i | x_i) p(x_i | y_{1:i-1}) dx_i$  is normalizing constant.

Equation 6 and 7 make up the fundament of Bayesian filtering. Since Equation 6 is not analytic especially for nonlinear problems, PF provides a nonparametric Monte Carlo method to realize recursive state estimate with a warm of particles  $x_i^{(j)}$  and their weights  $\omega_i^{(j)}, j=1, \dots, N$ . Then posterior pdf at  $i$  can be approximated as

$$p(x_i | y_{1:i}) = \sum_{j=1}^N \omega_i^{(j)} \delta(x_i - x_i^{(j)}) \quad (8)$$

and weights update as

$$\omega_i^{(j)} \propto \omega_{i-1}^{(j)} \cdot p(y_i | x_i^{(j)}) \quad j = 1, 2, \dots, N \quad (9)$$

According to the law of large numbers, Equation 8 approximates real posterior distribution when particles number  $N$  tends to infinity. Mentioned by Arulampalam et al. (2002), sample impoverishment and particle degeneracy exist in PF. Hence, Sequential importance sampling (SIS) and resampling are chosen.

### 3. Proposed method

Figure 2 shows our idea to deal with long-term RUL prediction based on ADT and PF with data sources from laboratory test and field operation.

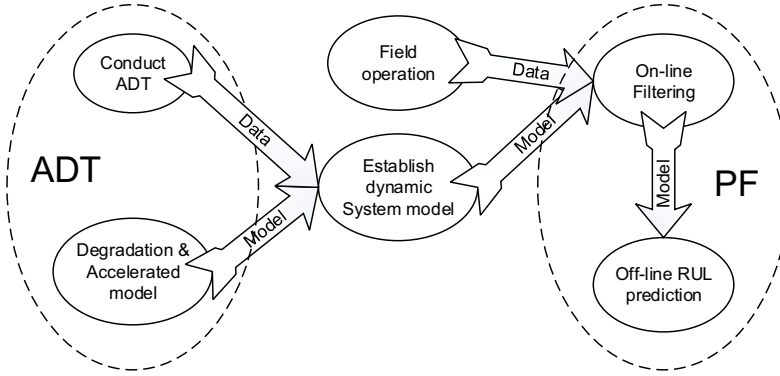


Figure 2: Graphical representation of the proposed method

Here, we consider that product degradation rate in normal condition and the monitor data acquired from instruments are system state and measurement, respectively. The proposed method follows such steps:

#### Step 1: Modelling degradation process

Assumed that ADT has  $k$  stress levels, i.e.  $s_1 < s_2 < \dots < s_k$ , and  $n$  samples. Time interval is  $\Delta t$  and measurement number of the  $i^{\text{th}}$  stress level is  $M_i$  where  $\sum_{i=1}^k M_i = M$ . Based on ADT data, unknown parameters  $[d(S_0), A, B, \sigma]$  in section 2.1 can be estimated by MLE (Ge et al., 2011). Thus, dynamic system model is established (Equation 2 and 3);

#### Step 2: On-line filtering

Based on the model of step 1, on-line filtering can be done through field data. In another words, we update particles and their weights by real usage data to ensure that particles reflect actual state of the product.

- 1) Initialize particles  $[x_0^{(j)}, \omega_0^{(j)} = 1/N, j=1, \dots, N]$  based on prior information, i.e.  $P(x_0)$ .
- 2) At time  $i$ , draw particle  $x_i^{(j)} \sim p(x_i | x_{i-1}^{(j)})$  and their weights  $\omega_i^{(j)} = p(y_i | x_i^{(j)})$ ,  $j=1, \dots, N$ . Set  $\omega_i^{(j)} = \omega_i^{(j)} / \text{SUM}(\omega_i^{(j)})$ .
- 3) Minimum mean square estimate of unknown state  $\hat{x}_i = \sum_{j=1}^N \omega_i^{(j)} x_i^{(j)}$ .
- 4) Resample to get new particles  $[x_i^{(j)}, \omega_i^{(j)} = 1/N, j=1, \dots, N]$ . See algorithm 2 in (Arulampalam et al., 2002).
- 5) Set  $i=i+1$ , turn back to 2) until  $i > t_{\text{On-line}}$ .

#### Step 3: Off-line RUL prediction

Special attention should be paid to off-line RUL prediction when no new measurement is available. This results in no more particles can be used for updating system model. Hence, RUL prediction algorithm based on PF can't be continued. To overcome this problem and realize long-term prediction, Orchard (2007) present three approaches. In consideration of computational burden and time-consuming problem, the third approach is proper for our purpose by considering particle weights invariant for future time instants which assumes particles are robust enough after on-line filtering. Therefore, when off-line RUL prediction goes on, we keep particle weights the same as the last moment of on-line filtering, expressed as

$$x_{t+i+1} = E[f(x_{t+i}^{(j)}, v_{t+i})] = E[x_{t+i+1}^{(j)}], \omega_{t+i+1}^{(j)} = \omega_t^{(j)} \quad j = 1, 2, \dots, N \quad (10)$$

where  $t$  is the total time of on-line filtering,  $x_{t+i+1}$  is the estimate state value at time  $t+i+1$ . Based on Equation 10, off-line filtering can be continued without new measurement. Future prediction is the weighted sum of current particles. In order to evaluate the effectiveness of our method, Root Mean Squared Error (RMSE) is introduced as

$$RMSE = \sqrt{\frac{1}{N-1} \sum_{j=1}^N \left( \frac{x_i^{(j)} - x_i}{x_i} \right)^2} \quad (11)$$

where  $x_i^{(j)}$  is the  $j^{\text{th}}$  particle at time  $i$ ,  $x_i$  is the measurement at time  $i$ ,  $N$  is the number of particles.

#### 4. Case study

Super Luminescent Diode (SLD) is a kind of light amplification device and has many characteristics like wide spectrum, high power spontaneous radiation, etc. Due to its advantages of wide spectrum and high output power, SLD is widely used in low coherent measurement system and special light sources. Since SLDs are designed as high-reliability and long-life for nowadays particular application, life-prediction for SLD is a tough task for engineering. Under the constraint of time, cost and the accuracy of prediction, SSADT is applied to gain sufficient degradation information under accelerated stress level.

In this case, temperature is the accelerated stress and Arrhenius model could be used as acceleration model. We monitored the output optical power of 3 SLDs and took record each 10 minutes. Table 1 shows the details. Here we use the concept of relative degradation quantity (Equation 12) to describe the deterioration status of SLD. Figure 3 shows the result of SSADT.

$$y_i = \frac{y'_0 - y'_i}{y'_0} \quad (12)$$

Where  $y'_i$  donates the output optical power at time  $i$ ,  $y'_0$  is the initial output.

Table 1: SSADT information of SLD ( $T$  represents Temperature)

	Normal( $T_0$ )	$T_1$	$T_2$	$T_3$	$T_4$
Stress level (Degree Celsius)	25	60	80	100	110
Number of measurement (Times)	(293320 min)	4519	4292	1806	2052

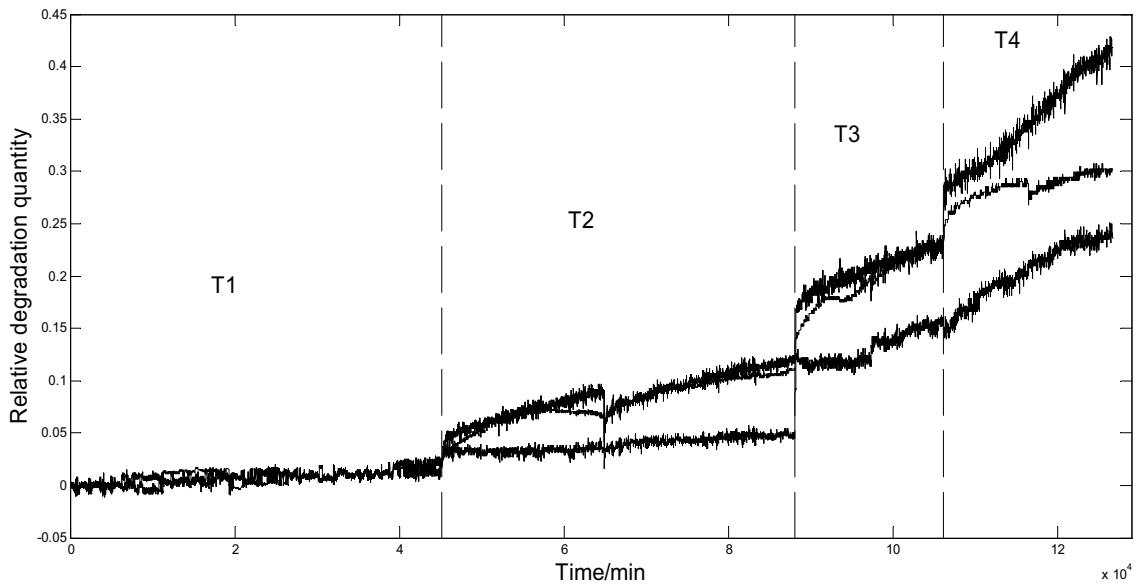


Figure 3: Results of SSADT: Relative degradation quantity of 3 samples.

For Arrhenius model  $d(T_i)=exp(A+B/T_i)$ , we take natural logarithm of both sides of the model, giving as  $ln(d(T_i))=A+B/T_i$ . Based on SSADT data, we can extrapolate degradation rate under normal condition (Meeker et al., 1998), i.e.  $d(T_0)$ . Also we can estimate other model parameters of Equation 2 and 3 by MLE, see Table 2.

Table 2: Model parameters estimate based on SSADT data

Parameter	$d(T_0)$	$A$	$B$	$\sigma$
Estimate value	2.1909e-08	6.7656	-7276	5.5471e-08

Therefore, we established state equation and measurement equation based on Equation 2 and 3. In addition to SSADT, operational data of SLD was also collected in service phase which is 293320 minutes in total. The monitor interval  $\Delta t$  is 10 minutes. Here we assume that error of degradation rate and measurement are nearly zero based on prior knowledge of experiment system, i.e. state error and measurement error are 0 and 0.0004, respectively. Then PF was used for long-term prediction with the  $N=1000$  particles. In order to validate the effectiveness of our method in section 3, operational data was divided into 2 parts: 10000 minutes for training and the rest for validation. In addition, off-line prediction by DBM (Equation 1) with SSADT data and regression with training data were also applied. See Figure 4.

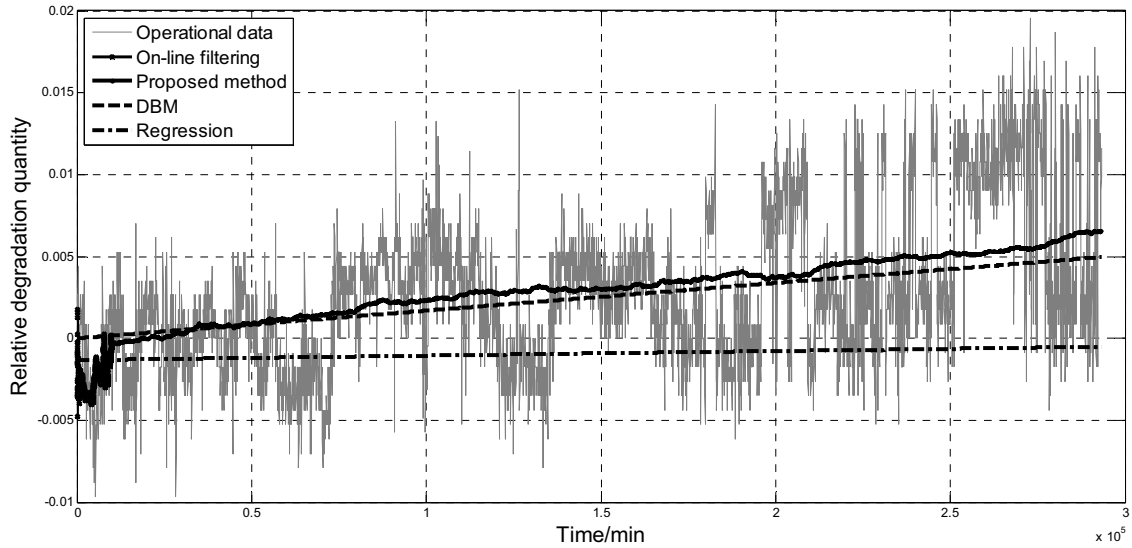


Figure 4: RUL prediction: proposed method, Regression and DBM.

In Figure 4, original data fluctuates seriously. Under the limitation of training data, the proposed method fit the real trend of degradation data well during the long-term period compared with regression method which seriously deviates the trend. Meanwhile, DBM fits well in RUL prediction. For quantification, RMSE of both on-line filtering and off-line RUL prediction for the proposed method are given in Figure 5.

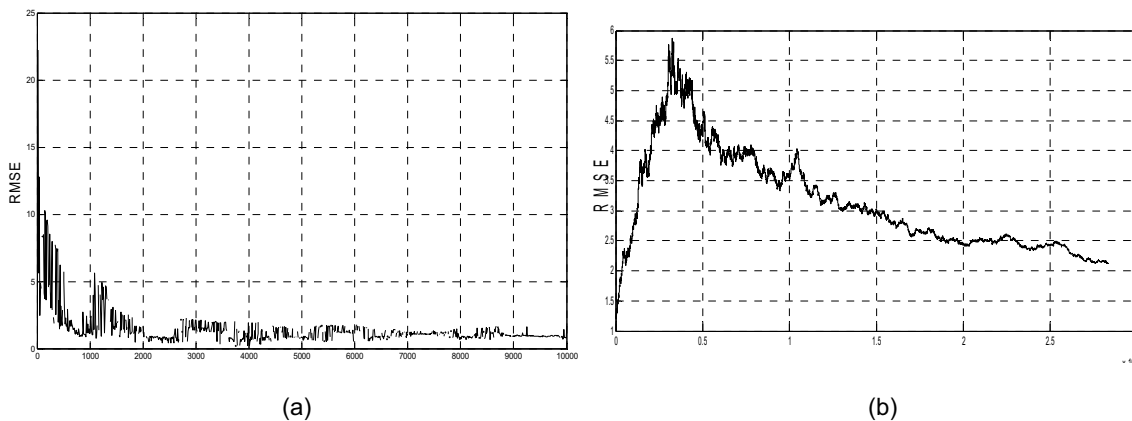


Figure 5: RMSE for the proposed method: On-line ((a), 0 to 10000 min), Off-line ((b), 10001 to 293320 min)

Summaries are given as:

- i) Figure 5 (a) significantly shows that RMSE of on-line filtering stage rapidly decline to a low and stable level which represents the robustness of particles after on-line filtering. And it also verifies the former assumption that particle weights are robust enough to conduct off-line prediction.
- ii) Figure 5 (b) shows that RMSE increase at the beginning and then decline to a low level. Increasing of RMSE is caused by the initial increment of particles' value, i.e., the increment of degradation. Along with the RUL prediction, the influence of the initial particles becomes negligible and then RMSE declines to an acceptable level.

Taken (a) and (b) together, we can see that our proposed method can well handle the case of integrating ADT data with operational data and make accurate prediction.

## 5. Conclusions

In this work, a method based on accelerated degradation testing and particle filtering has been introduced for RUL prediction of high-reliability and long-life products. The combination of infield ADT which is widely used in R&D phase and field operation can not only comprehensive evaluate products' health state but also improve the credibility and correctness of prediction results while saving time and cost. In our work, ADT is used for gathering degradation information of products in a shorter time than that experienced in normal condition. Based on that, dynamic system model can be established by stochastic process to describe degradation process. Two stages of RUL prediction then can be made by on-line filtering and off-line prediction. The introduced index RMSE and the real application of SLD verify the effectiveness of the proposed method.

In our research, we assume drift coefficient, i.e. degradation rate in ADT, is constant. Future research need to consider it as time-dependent random variable represented by Si et al. (2013). Also another stochastic process, i.e., Gamma process, should be taken into consideration.

## References

- Arulampalam M.S., Maskell S., Gordon N., Clapp T., 2002, A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking, *IEEE Transactions on Signal Processing*, 50, 174-188.
- Chen C., Vachtsevanos G., Orchard M.E., 2012, Machine remaining useful life prediction: An integrated adaptive neuro-fuzzy and high-order particle filtering approach, *Mechanical Systems and Signal Processing*, 28, 597-607.
- Escobar L.A., Meeker W.Q., 2006, A review of accelerated test models, *Statistical Science*, 21, 552-577.
- Ge Z.Z., Li X.Y., Jiang T.M., Huang T.T., 2011, Optimal design for step-stress accelerated degradation testing based on D-optimality, *Annual Reliability and Maintainability Symposium Lake Buena Vista, FL, United states: Institute of Electrical and Electronics Engineers Inc.*, DOI: 10.1109/rams.2011.5754481.
- Li X.Y., Jiang T.M., Sun F.Q., Ma J., 2010, Constant Stress Adt for Superluminescent Diode and Parameter Sensitivity Analysis, *Eksploatacja I Niezawodnosc-Maintenance and Reliability*, 21-26.
- Meeker W.Q., Escobar L.A., Lu C.J., 1998, Accelerated degradation tests: modeling and analysis, *Technometrics*, 40, 89-99.
- Orchard M.E., 2007, A particle filtering-based framework for on-line fault diagnosis and failure prognosis. Ph.D., Georgia Institute of Technology.
- Si X., Wang W., Hu C., Zhou D., 2011, Remaining useful life estimation – A review on the statistical data driven approaches, *European Journal of Operational Research*, 213, 1-14.
- Si X., Wang W., Hu C., Chen M., Zhou D., 2013, A Wiener-process-based degradation model with a recursive filter algorithm for remaining useful life estimation, *Mechanical Systems and Signal Processing*, 35, 219-237.
- Sikorska J.Z., Hodkiewicz M., Ma L., 2011, Prognostic modelling options for remaining useful life estimation by industry, *Mechanical Systems and Signal Processing*, 25, 1803-1836.
- Tang L.C., Yang G., Xie M., 2004, Planning of step-stress accelerated degradation test, *Reliability and Maintainability, 2004 Annual Symposium-RAMS: IEEE*, 287-292.
- Zio E., Peloni G., 2011, Particle filtering prognostic estimation of the remaining useful life of nonlinear components, *Reliability Engineering & System Safety*, 96, 403-409.
- Zio E., Broggi M., Golea L.R., Pedroni N. 2012, Failure and reliability predictions by infinite impulse response locally recurrent neural networks, *Chemical Engineering Transactions*, 26, 117-122.