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RUL Estimation Using an Adaptive Inverse Gaussian Model

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In this paper, an adaptive inverse Gaussian stochastic process is developed to characterize the degradation process of condition monitored components. The knowledge of the degradation process is updated through the parameter of the process when new observations are available. The updating is performed through a general Bayesian filtering process within a state space model setting. The proposed adaptive model is history-dependent and can adjust itself to the sudden changes in degradation signals. The numerical case study shows that the variance of the RUL distribution obtained from the adaptive model is less than that of the conventional inverse Gaussian model and the predictive accuracy is improved by using the adaptive model in terms of TMSE. To validate our adaptive model further, we conduct a model prediction accuracy test. Our test result enables us to conclude that our model is stable, robust and beneficial for the application in prognostics and health management of systems.

1. Introduction

With the appearance of large industrial systems covering wide areas or large populations, such as electrical power systems, marine and aviation systems, prognostics has become increasingly important for optimising the operation of these systems and their critical components in terms of higher reliability, less cost, higher production quality, better inventory planning and more effective reuse of industrial resources (Jardine et al. (2006)). A timely and correct prognostic method could assist the selection of effective condition based strategies for industrial purposes, especially for vital or critical systems (Pecht (2008)). An effective prognostic strategy usually involves the accurate prediction of residual useful life (RUL) for a system under investigation.

Stochastic processes have been used widely for modelling degradation because of uncertainty relating to the nature of system deterioration (Engel et al., 2000). The RUL of such a system can be solved through finding the first passage time (FPT) of the degradation process. The inverse Gaussian (IG) process is best known as corresponding to the first passage time of a Brownian motion with drift. Nevertheless, the utilization of the IG process to model degradation has not drawn much attention. Only Wang and Xu (2010) appeared to use an IG process to describe component degradation for the purpose of time to failure prediction. However, in their paper, they considered predicting the RUL using only current degradation information and did not investigate the impact of past CM information on the prediction of RUL. This issue has been addressed both in Si et al. (2011a) and Wang et al. (2011), but not in the context of IG processes. However they have demonstrated that using both past and current CM information is critical to the accuracy of prognosis. Therefore, we believe it is necessary to make the IG degradation process adaptive to past information for the purpose of RUL prognosis.

2. Adaptive degradation modelling using an inverse Gaussian process

2.1 Inverse Gaussian process

An inverse Gaussian process is a stochastic process with independent inverse Gaussian distributed increments . We now define an inverse Gaussian process as described in Chhikara and Folks (1989), with mean parameter bt and scale parameter λt^2, as a stochastic process $\{X_t, t \geq 0\}$ with the following properties:

1. $X_0 = 0$ with probability one;
2. X_t has independent increments, so that for every pair of disjoint intervals (t_1, t_2) and (t_3, t_4) with $t_1 < t_2 < t_3 < t_4$, the random variables $X_{t_2} - X_{t_1}$ and $X_{t_4} - X_{t_3}$ are independent;
3. Each increment $X_\tau - X_t$ has an inverse Gaussian distribution with mean $b(\tau - t)$ and scale parameter $\lambda(\tau - t)^2$, for all $\tau > t$.

Let L be the threshold of an inverse Gaussian process, and suppose that we are interested in the distribution of the time T that the degradation first exceeds L, assuming it starts from 0. Here we use $F_T(t)$ to denote the distribution function of FPT. Since it is a non-decreasing process, the definition of FTP $T = \inf\{t : X_t \geq L\}$ is equivalent to $T = \{t : X_t \geq L\}$. It follows that the distribution of FPT for an inverse Gaussian process is derived in the following way (Wang and Xu, 2010),

$$F_T(t) = \mathrm{P}(X_t \geq L) = \Phi\left(-\sqrt{\frac{\lambda}{L}}\left(\frac{L}{b} - t\right)\right) - \exp\left(\frac{2\lambda t}{b}\right)\Phi\left(-\sqrt{\frac{\lambda}{L}}\left(\frac{L}{b} + t\right)\right) \quad (1)$$

where $\Phi(\cdot)$ is the CDF of standard normal distribution. By differentiating Equation (1) with respect to t, we obtain a formula for the PDF of the first passage time T.

2.2 Adaptive degradation modelling

An inverse Gaussian process Γ_t with mean parameter $b_t = b_{t_i} t$ and scale parameter λt^2 can be used for describing the evolution of the degradation based on obtained condition monitoring (CM) information. We use some additional notation: m is the number of CM points before failure; t_i is the time of the ith CM point; Y_t is a random variable representing the CM information at time t; y_t is the observed realisation of Y_t; $\mathbf{Y}_{t_i} = \{y_{t_0}, y_{t_1}, ..., y_{t_i}\}$ is the history of degradation observations for the plant up to time t_i. At each CM point t_i, we observe the current degradation observation y_{t_i}. The future degradation y_t , $t > t_i$ is regarded as a three-parameter inverse Gaussian random variable with location parameter y_{t_i}, mean parameter $b_{t_i}(t - t_i)$ and scale parameter $\lambda(t - t_i)^2$. The probability density function of y_t is given as.

$$p(y_t \mid y_{t_i}; b_{t_i}, \lambda) = \sqrt{\frac{\lambda(t - t_i)^2}{2\pi v_{t_i}^{\ 3}}} \exp\{-\frac{\lambda(t - t_i)^2 v_{t_i}}{2}(b_{t_i}(t - t_i) - \frac{1}{v_{t_i}})^2\} \quad (2)$$

2.3 Updating the mean parameter

From section 2.2, it is noted that b_{t_i} is not a constant and regarded as a random variable. Therefore, we construct a state space model for the evolvement of b_{t_i}. In this state space setting, b_{t_i} can be updated through a Bayesian filtering process at each CM point once new information is obtained. It is worth noting that the updating process for b_{t_i} is difficult as an analytical form is not generally available. As such, we make a few assumptions and attempt to obtain a closed form for b_{t_i}. First, we assume that CM information is collected based on a fixed time interval. This is a mild and reasonable assumption as, in practice, fixed time sampling is widely used and is also convenient. For notational simplicity, we will use

Δt to denote the equal interval. The increment v_{t_i} is the value between the current checking point y_{t_i} and the last point $y_{t_{i-1}}$, $v_{t_i} = y_{t_i} - y_{t_{i-1}}$. In the following work, we use $\mathbf{V}_{t_i} = \{v_{t_1}, v_{t_2}, ..., v_{t_i}\}$ as the observed information which is equivalent to $\mathbf{Y}_{t_i} = \{y_{t_0}, y_{t_1}, ..., y_{t_i}\}$. According to the properties of an inverse Gaussian process, at each CM point t_i, the increment follows an inverse Gaussian distribution, that is $p(v_{t_i} | b_{t_i}) = IG(v_{t_i}; b_{t_i}\Delta t, \lambda\Delta t^2)$. The mean parameter b_{t_i} before considering the observed y_{t_i} is assumed to satisfy $b_{t_i} = b_{t_{i-1}}$. Then the semi-deterministic state space equation (Equation (3)) is constructed for the updating of b_{t_i}. Other state space equations can also be considered, and the reasons we choose this particular type lie in: 1). An explicit solution can be obtained in this model setting; 2. it is reasonable to assume that in between two checking points, the mean parameter does not change dramatically. In other cases, a particle filtering method is needed to get an approximate solution.

$$\begin{cases} b_{t_i} = b_{t_{i-1}} \\ p(v_{t_i} | b_{t_i}) = \sqrt{\dfrac{\lambda\Delta t^2}{2\pi v_{t_i}{}^3}} \exp\{-\dfrac{\lambda\Delta t^2 v_{t_i}}{2}(b_{t_i}\Delta t - \dfrac{1}{v_{t_i}})^2\} \end{cases} \tag{3}$$

Then, under a Bayesian filtering process, we have from Equation (3) that

$$p(b_{t_i} | \mathbf{V}_{t_i}) = p(b_{t_i} | v_{t_i}, \mathbf{V}_{t_{i-1}}) = \frac{p(v_{t_i} b_{t_i} | \mathbf{V}_{t_{i-1}})}{p(v_{t_i} | \mathbf{V}_{t_{i-1}})} = \frac{p(v_{t_i} | b_{t_i}) p(b_{t_i} | \mathbf{V}_{t_{i-1}})}{\int_0^\infty p(v_{t_i} | b_{t_i}) p(b_{t_i} | \mathbf{V}_{t_{i-1}}) dk_{t_i}} \tag{4}$$

Recursively applying the Bayes' rule, we have the probability distribution of b_{t_i} conditional on the collected information up to time t_i

$$p(b_{t_i} | \mathbf{V}_{t_i}) = \frac{p(v_{t_i} | b_{t_i}) p(v_{t_{i-1}} | b_{t_i}) ... p(v_{t_1} | b_{t_i}) p(b_{t_i} | V_0)}{\int_0^\infty p(v_{t_i} | b_{t_i}) p(v_{t_{i-1}} | b_{t_i}) ... p(v_{t_1} | b_{t_i}) p(b_{t_i} | V_0) dk_{t_i}} \tag{5}$$

As $b_{t_i} = b_{t_{i-1}}$ is assumed, $p(b_{t_i} | V_0) = p(b_{t_{i-1}} | V_0) = p(b_{t_0} | V_0)$ holds. This largely decreases the computational burden of the estimation process and this is another reason why we choose the deterministic form $b_{t_i} = b_{t_{i-1}}$. Then the distribution of $p(b_{t_0} | V_0)$ is essential to get an explicit solution of Equation (5). In Banerjee and Bhattacharyya (1979) and Seshadri (1993), it has been established that if the prior distribution of a random variable follows a truncated normal distribution, left truncated at 0, then the posterior distribution should be from the gamma family. Now we assume that the distribution of $p(b_{t_0} | V_0)$ follows a left truncated normal distribution with parameters $1/\beta_0$ and $1/\tau_0$:

$$p(b_{t_0} | V_0) = \frac{\dfrac{1}{\sqrt{2\pi/\tau_0}} e^{-\frac{(b_{t_0} - 1/\beta_0)^2}{2/\tau_0}}}{\int_0^\infty \dfrac{1}{\sqrt{2\pi/\tau_0}} e^{-\frac{(b_{t_0} - 1/\beta_0)^2}{2/\tau_0}} db_0} = \frac{e^{-\frac{(b_{t_0} - 1/\beta_0)^2}{2/\tau_0}}}{\int_0^\infty e^{-\frac{(b_{t_0} - 1/\beta_0)^2}{2/\tau_0}} db_0} \tag{6}$$

Substituting the PDF of $p(b_{t_0} | V_0)$ into Equation (5), we have the distribution of the mean parameter b_{t_i} detailed in equation (7) through several manipulations:

$$p(b_{t_i} | \mathbf{V}_{t_i}) = \frac{\exp\{\frac{-\tau_i(b_{t_i} - 1/\beta_i)^2}{2}\}}{\int_0^\infty \exp\{\frac{-\tau_i[b_{t_i} - 1/\beta_i]^2}{2}\} db_{t_i}} \tag{7}$$

where $\tau_i = \lambda\Delta t^2 \sum_{j=1}^{i} v_{t_j} + \tau_0$, $1/\beta_i = \frac{i\Delta t + \tau_0/(\lambda\beta_0)}{\Delta t^2 \sum_{j=1}^{i} v_{t_j} + \tau_0/\lambda}$.

For simplicity, we let $\tau_0 = \alpha\lambda\beta_0$, then the parameters β_i and τ_i in the PDF $p(b_{t_i} | \mathbf{V}_{t_i})$ can be expressed in terms of parameters λ, α and β_0 as.

$$\beta_i = \frac{\Delta t^2 \sum_{j=1}^{i} v_{t_j} + \alpha\beta_0}{i\Delta t + \alpha}, \qquad \tau_i = \lambda\Delta t^2 \sum_{j=1}^{i} v_{t_j} + \alpha\lambda\beta_0 \tag{8}$$

Having the density function of $p(b_{t_i} | \mathbf{V}_{t_i})$ available, we can substitute the mean of b_{t_i} into Equation (1) in order to get the time to failure or RUL at each monitored point. Alternatively, we could utilize the whole distribution of $p(b_{t_i} | \mathbf{V}_{t_i})$ for the calculation of RUL as discussed in Si et al. (2013).

3. Numerical example

In this section, we provide a numerical example of crack growth to demonstrate the performance of an adaptive inverse Gaussian process based degradation model and the conventional inverse Gaussian model. The crack growth data are from Lu and Meeker (1993).The expectation maximization (EM) algorithm (Dempster et al., 1977) is applied for parameter estimation as hidden variable is used in our model. The parameters in the conventional inverse Gaussian model are estimated using the maximum likelihood estimation algorithm (Kendall and Alan, 1973). The estimated parameters for both models are presented in Table 1.

Table 1 Estimated parameter values for both models

Parameter	Adaptive model	Parameter	Conventional model
α	31.6860	b	0.0602
β_0	61.7136		
λ	0.1863	λ	0.3240

Using the estimated increments, the tracking ability of the adaptive model can be demonstrated. Figure 1 compares the length of a real crack and the predicted results of the crack length. It can be seen that the predicted results from the adaptive model are closer to the real data. It should be noted that the real crack length increases at a higher rate after check time eight. Since the adaptive model adjusts its parameters from new observed information, the new crack length information is utilized in the model to update the parameters for future prediction after each check time. From Figure 1, it is shown that the estimated crack length agrees with the suddenly increasing rate of the real crack growth at check point eight and the adaptive model quickly catches up with this change, by adjusting itself to follow the real situation.

Using the estimated parameters, we can also derive the PDF of the failure time or RUL using Equation (1). For comparison purposes, we plot the RUL PDFs for both models, the adaptive inverse Gaussian based model and the conventional inverse Gaussian based model, in Figure 2. The real RULs are lying in 95 % confidence interval of RUL distribution obtained from the adaptive model at each CM point. While for the

RUL distribution obtained from conventional model, the real RUL at the last CM point falls out of the 95 % confidence interval. In addition, the real RULs are noticeably much closer to the modes of the RUL distributions obtained from the adaptive model as time progresses. Furthermore, the variances of the RULs produced by the adaptive model are apparently smaller than those of the conventional model, which shows that the predictive accuracy of the former is better than that of the latter.

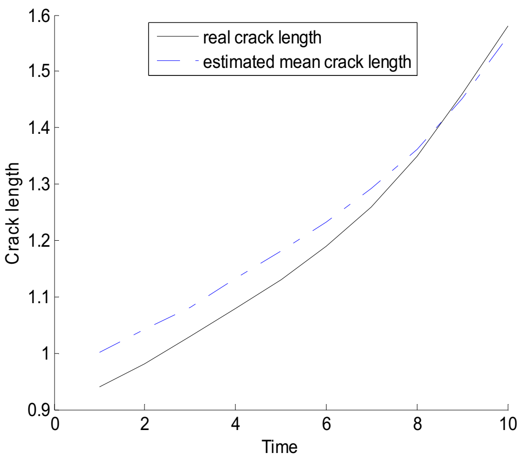

Figure 1 Crack length prediction

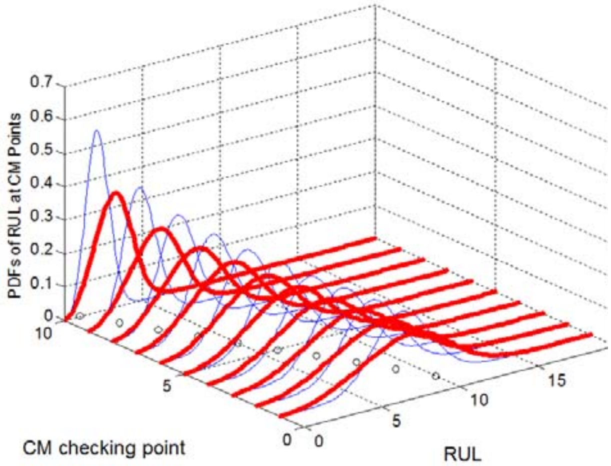

Figure 2 Residual life PDFs of adaptive inverse Gaussian model (blue) and conventional inverse Gaussian model (red bold)

In addition, we compare, for each testing data group, the probabilities of late prediction of both models in Table 2, which are the probabilities of predicting the RUL later than the real RUL. In asset management, late predictions might cause catastrophic consequences and late alarms will be punished heavily. Thus the probability of late prediction is usually used as a popular criterion for many safety critical systems. A smaller probability of late prediction commonly ensures a higher reliability of the system.

Table 2 Average late prediction probability

Average late prediction probability	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Adaptive inverse Gaussian	0.5949	0.5785	0.5722	0.5781	0.5704	0.5714
Conventional inverse Gaussian	0.6018	0.5837	0.5786	0.5839	0.5784	0.5768

From Table 2, we note that the difference between total late prediction probabilities produced from both models is not great, but the outcomes from the conventional inverse Gaussian process based model are still slightly larger than those from the adaptive inverse Gaussian model for all the six groups of testing data.

Next, the total mean square errors (TMSE) of these two models are calculated and the results are presented in Table 3. TMSE is a key criterion for model comparison and selection, and a model with smaller TMSE is usually regarded as a better model. The results show that the TMSE of the adaptive inverse Gaussian model is about 35 % smaller than that of the conventional inverse Gaussian model, which provides further evidence in support of the conclusion that the proposed adaptive model is better than the conventional model.

Table 3 TMSE comparison

Model	TMSE
Adaptive inverse Gaussian model	321.1910
Conventional inverse Gaussian model	498.6807

To validate our model further, we conduct a model prediction accuracy test. We choose six groups at random from the twelve available; then we train the six groups and test with the other six groups. The process is repeated a further three times, so that we conduct a total of four independent experiments.

Throughout the parameter estimation and testing procedures for both the adaptive Gaussian model and the conventional Gaussian model, we get the prediction results shown in Table 4. By comparing these model outputs, we can assess whether the model is sensitive to the data change. From Table 4, we can see that our model performs reasonably consistently for the given different datasets and outperforms the conventional Gaussian model respectively in terms of TMSE. Therefore, our prediction accuracy test enables us to conclude that our model is stable, robust and beneficial for the analysis of different datasets.

Table 4 TMSE results of four trials for conventional and adaptive Gaussian models

Model TMSE	Trial 1	Trial 2	Trial 3	Trial 4
Adaptive Gaussian model	276.9428	269.572	343.4941	290.4350
Conventional Gaussian model	451.907	449.7512	511.4516	468.3932

4. Conclusions

In this paper, an adaptive inverse Gaussian stochastic process is developed to characterize the degradation process of monitored components. The knowledge of the degradation process is updated through the mean parameter of the inverse Gaussian process when new observations are available. The updating is performed through a general Bayesian filtering process within a state space model setting. The proposed adaptive model is history-dependent and could adjust itself to the sudden changes in degradation signals. The numerical case study shows that the variance of the RUL distribution obtained from the adaptive model is less than that of the conventional inverse Gaussian model and the predictive accuracy is improved by using an adaptive model in terms of TMSE. Furthermore, a model prediction accuracy test is carried out to validate our model. The results show that the adaptive Gaussian model is robust and can be applied to different datasets with desirable results.

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