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Sequential Inspection and Maintenance Policy for Multiple Systems with Wiener Degradation Failure under Availability Requirement

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This paper investigates the sequential inspection and maintenance policy for multiple systems under availability requirement. The state of each system is described by an identical Wiener degradation process with unknown parameters. A system failure is defined by the excess of degradation to certain critical value. The degradation of each system can only be known through non-periodic, discrete time inspection. At each inspection time, preventive maintenance or replacement is carried out considering the degree of system degradation. There are two types of critical values, alarm threshold and failure threshold, both of which are used to determine the maintenance actions combined with the system degradation history. The unknown parameters are sequentially estimated incorporated into the decision making, and then the next inspection time is determined under average availability requirement. Different alarm thresholds are analysed and the proposed inspection and maintenance procedure is illustrated with numerical examples.

1. Introduction

We consider the problem of inspection and maintenance policy for the case when there are several similar systems operating simultaneously on the site or in storage. Each system can be a single component or a more complex system whose state is described by a Wiener degradation process X_t. System failures occur when X_t goes beyond certain critical value. The degradation of the system may actually be usually unknown unless it is inspected. Although continuous monitoring and inspection is possible, periodic or discrete time inspection is usually employed in practice, due to the cost and other constrictions.

For system whose degree of degradation is only detected at the time of inspection, it is important to determine the optimal time of inspection. Fewer inspections will lead to lower reliability, while frequent inspection will lead to higher cost. When there is a reliability requirement, the problem is usually to develop an inspection and maintenance policy that meets these requirements. The problem is formulated for periodic inspection to minimize the cost with respect to the time interval for inspection. For example, Cerone (1993) investigated the optimal inspection interval for maximum future reliability using the delay-time model. Ito and Nakagawa (2000) studied the optimal periodic inspection policies for a system in storage with degradation. Hariga (1996) discussed a maintenance and inspection model for a single machine with general failure distribution. Vaurion (1999) studied the availability and cost functions in the situation of periodic inspection and preventive maintenance for multiple units. Most of the optimal policies based on average cost are derived via asymptotical theory. In general, it requires a long period of usage before the system reaches the limiting or steady-state, and usually it is not clear whether the asymptotic results are accurate enough.

Yang and Klutke (2000) studied some inspection policies, in which they focused on steady-state availabilities for several models. The inspection policy defined is based on the required availability of the system. Lam and Yeh (1994) discussed a sequential inspection policy and compared it with some continuous strategies based on a finite-state continuous-time Markov model, and similar research is referred to Yeh (1997). Chelbi and Ait-Kadi (2000) also considered some general inspection policies.

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Cui et. al. (2004) investigated a sequential inspection strategy for multiple systems under different availability criteria constrictions. In their paper, with replacement or perfect repair assumption, the unknown system lifetime distribution is incorporated into the analysis and decision making.

In this paper, we develop a sequential inspection and maintenance policy when there are a number of similar deterioration systems operating on the same environment or in storage. Specially, we assume that the degradation path of each system follows an identical Wiener processes, but parameters are unknown. Each system is inspected at discrete time, and actions such as preventive maintenance and replacement are taken based on the degradation measured at each inspection time. There are two types of critical values, failure threshold D_f and alarm threshold D_a. If the system degradation exceeds D_f, then the system is replaced by a new one. If the degradation is smaller than D_f, but larger than D_a and the system has never been maintained, then an preventive maintenance is carried out. If the degradation is larger than D_a and the system has been maintained once, then a replacement is carried out even though the degradation is smaller than D_f. On other conditions, the system will operate without any maintenance until the next inspection. The time to the next inspection is determined based on the results of previous inspections. Furthermore, parameters estimation is incorporated into the analysis. The average availability criterion is used and different alarm thresholds are analyzed. The proposed procedure is illustrated with numerical examples.

2. The model and assumptions

We assume that there are *n* systems in the field and they are inspected at the same, but discrete times. The following assumptions are used.

1. The degradation paths of all systems follow identical Wiener processes, but the parameters are unknown.
2. The inspections are carried out at times τ_1, τ_2, … ; and all *n* systems are inspected each time.
3. System degradation is observed only by inspection, and preventive maintenance or replacement is carried out depended on the degradation.
4. Preventive maintenance is imperfect. After each preventive maintenance, the system degradation decreases in proportion to the amount just before maintenance. Denote the proportional coefficient by p ($0 < p < 1$) which is known. Specially, we assume that each system undergoes such preventive maintenance at most one time before it failures.
5. The inspection action does not intervene with the system degradation, and the inspection time and preventive maintenance or replacement time are negligible.
6. The degradation process continues even though it has exceeded D_f, and this assumption avoids troublesomely truncated situation during estimating parameters.

The required availability of the systems is denoted by R_s. One degradation path of the inspected and maintained system is shown in *Figure 1*.

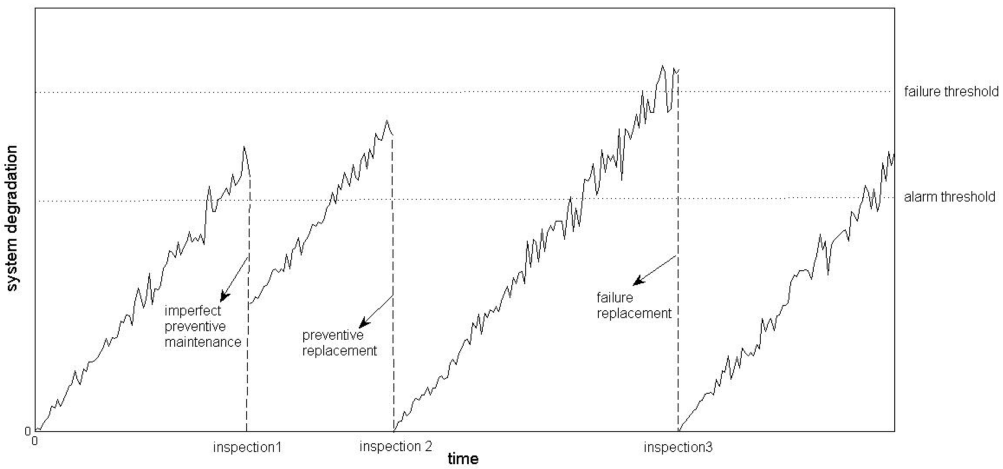

Figure 1: the degradation path of one inspected and maintained system

Let $D_i(t)$ represent the degradation path of the *i*th system. Then,

$$D_i(t) = \begin{cases} \eta t + \sigma B(t), & t < T_p \\ p[\eta T_p + \sigma B(T_p)] + \eta(t - T_p) + \sigma B(t - T_p), & T_p \le t < T_r \\ \eta(t - T_r) + \sigma B(t - T_r), & t \ge T_r \end{cases} \quad (1)$$

where t denotes operation time, T_p denotes the preventive maintenance time, T_r denotes the replacement time, $B(\cdot)$ is a standard Brownian motion, η is the drift rate, $\eta > 0$, σ is the diffusion coefficient.

As the degradation path follows a Wiener process, the lifetime distribution is an inverse Gaussian distribution. Denote by $D_i^{(k)+}$ the system degradation at the instant just after the *k*th inspection (an preventive maintenance or replacement may be carried out at this inspection time), the probability that the system is still functioning at time t ($t > \tau_k$) is given by:

$$R_i(t) = 1 - \Phi\left(\frac{-(D_f - D_i^{(k)+}) + \eta(t - \tau_k)}{\sigma\sqrt{t - \tau_k}}\right) - \exp\left(\frac{2\eta(D_f - D_i^{(k)+})}{\sigma^2}\right)\Phi\left(\frac{-(D_f - D_i^{(k)+}) - \eta(t - \tau_k)}{\sigma\sqrt{t - \tau_k}}\right) \quad (2)$$

where $\Phi(\cdot)$ denotes cumulative distribution function (CDF) of a standard normal distribution with a zero mean and variance 1. When the parameters are known and there is only one system *i*, the next inspection time is $\tau_{k+1}^{(i)}$ which is the solution of $R_i(\tau_{k+1}^{(i)}) = R_s$.

When adopting a sequential strategy in practice, the parameters η and σ is usually unknown. Denote by $\hat{\eta}^{(k)}$ and $\hat{\sigma}^{(k)}$ the parameters estimators at inspection time τ_k , and the estimated survival function of the system at time t ($t > \tau_k$) is given by $\hat{R}_i(t)$. Then, the (*k*+1)th inspection time for system *i* is estimated as $\tau_{k+1}^{(i)}$ which is the solution of $\hat{R}_i(\tau_{k+1}^{(i)}) = R_s$.

In this paper, the average availability criterion is used to determine the the next inspection time in consideration of multiple systems: $\tau_j = \frac{1}{n}\sum_{i=1}^{n}\tau_j^{(i)}$. This criterion is a reasonable inspection and maintenance strategy which takes a balance among all the systems as a whole.

3. Sequential inspection and maintenance procedure

The main tasks for the sequential inspection and maintenance policy are:

(1) to estimate the unknown parameters of the Wiener degradation process;

(2) to derive optimal inspection and maintenance times, and take appropriate maintenance actions based on degradation information collected in the sequence of previous inspection times.

The sequential inspection and maintenance procedure is first stated as follows. The explanations are presented later.

3.1 The procedure

The sequential inspection and maintenance procedure can be summarized as follows:

Step 1: The first inspection time τ_1 is specified based on some other information or standard guidelines;

Step 2: After the first inspection at time τ_1 , the degradation of each system is measured and the degradation vector is denoted by $\{D_1^{(1)-}, \ldots, D_n^{(1)-}\}$;

Step 3: Estimate the parameters based on the sample $\{D_1^{(1)-}, \ldots, D_n^{(1)-}\}$;

Step 4: Determine the type of maintenance actions taken on each system based on its degradation. For system *i*, if $D_i^{(1)-} \ge D_f$, then the system is replaced by a new one. At the instant just after replacement, the degradation of system *i* is denoted by $D_i^{(1)+}$, and $D_i^{(1)+} = 0$. If $D_a \le D_i^{(1)-} < D_f$ and the system has never been maintained, then a preventive maintenance is carried out and $D_i^{(1)+} = pD_i^{(1)-}$. if $D_i^{(1)-} \ge D_a$ and the system has undergone preventive maintenance, then replacement is carried out . On other conditions, no actions are taken on the system, and $D_i^{(1)+} = D_i^{(1)-}$;

Step 5: Determine the next inspection time based on the average availability criterion and parameters estimator;

Step 6: Summarize the degradation information as M_k = {degradation information obtained by time τ_k } = $\bigcup_{j=1,2,\cdots,k}$ {the degradation vector of n systems is $\left\{D_1^{(j)-} - D_1^{(j-1)+},\ldots,D_n^{(j)-} - D_n^{(j-1)+}\right\}$ during interval $\left[\tau_{j-1},\tau_j\right]$} where $D_i^{(0)+} = 0$ ($i = 1,2,\ldots,n$) and $\tau_0 = 0$. Here M_k denotes all the degradation information on the whole systems collected after the kth inspection and maintenance.

3.2 Parameter estimation

Set $\Delta d_i^{(j)} = D_i^{(j)-} - D_i^{(j-1)+}$ and $\Delta\tau_j = \tau_j - \tau_{j-1}$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,k$, then, for $1 \le i \le n$, $1 \le j \le k$, $(\Delta d_i^{(j)}|\eta,\sigma)$ are independently distributed as $N(\eta\Delta\tau_j,\sigma\sqrt{\tau_j})$. Thus, given the degradation information M_k , the joint likelihood function at time τ_k can be expressed as follows:

$$l(\eta,\sigma)=\prod_{i=1}^{n} H_i(\eta,\sigma) \tag{3}$$

where

$H_i(\eta,\sigma)=\prod_{j=1}^{k} \frac{1}{\sqrt{\sigma^2\Delta\tau_j}}\phi\left(\frac{\Delta d_i^{(j)}-\eta\Delta\tau_j}{\sqrt{\sigma^2\Delta\tau_j}}\right)$ is the likelihood function corresponding to the system i based on the degradation data collected up to the kth inspection, and $\phi(\cdot)$ is the probability density function of the standard normal distribution. Thus, the estimates of η and σ can be obtained by maximizing $l(\eta,\sigma)$ or its logarithmic form directly.

4. Some illustrative examples

Here some numerical examples are shown in order to illustrate how the procedure works. The degradation data is simulated from the Wiener process $X(t)=\eta t+\sigma B(t)$, η =5, σ =3. The number of systems in the field is ten, and the required availability is R_s =0.8. The failure threshold D_f is 100, and the preventive maintenance proportional coefficient p =0.5. We shall discuss the results with respected to four different alarm thresholds: D_a =50, 60, 70, 80.

Table 1: Data from inspections and maintenances and the sequential inspection time (D_a =50)

No.	1		2		3		4		5		6		7		8	
	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+
1	55.10	27.55	60.88	0	71.31	35.66	103.60	0	Data omitted							
2	67.40	33.70	106.73	0	51.13	25.57	72.57	0								
3	28.57	28.57	80.69	40.35	112.15	0	47.80	47.80								
4	58.18	29.09	74.07	0	81.61	40.81	91.28	0								
5	53.02	26.51	78.53	0	69.35	34.68	63.19	0								
6	37.59	37.59	81.28	40.64	115.85	0	73.60	36.80								
7	45.89	45.89	90.32	45.16	117.07	0	62.15	31.08								
8	53.25	26.63	85.67	0	60.84	30.42	81.45	0								
9	83.95	41.98	100.29	0	67.26	33.63	106.53	0								
10	76.27	38.14	96.52	0	55.64	27.82	69.01	0								
$\Delta\tau_k$	10		9.11		12.82		11.76		14.10		11.95		14.72		12.16	
$\hat{\eta}^{(k)}$	5.59		5.65		5.52		5.30		5.27		5.19		5.19		5.14	
$\hat{\sigma}^{(k)}$	5.31		4.48		3.85		3.95		3.53		3.35		3.25		3.34	
N_1	7		10		17		19		26		27		34		37	
N_2	0		7		10		17		20		27		30		37	

Note that: throughout this section: '-' denotes the instant just before inspection; '+' denotes the instant just after inspection and maintenance; N_1 denotes the total preventive maintenance times; N_2 denotes the total replacement times.

For the case of D_a =50, the steps can be described as follows.

Step 1: Assume that based on other information we first determine $\Delta\tau_1$ =10. After the inspection and maintenance the simulated data shown is in the second and third columns of Table 1. From that we have M_1, then based on the formula in Section 3.2, we get $\hat{\eta}^{(1)}$ =5.59, $\hat{\sigma}^{(1)}$ =5.31. For i =1,… , 10, $\tau_2^{(i)}$ is the solution of $\hat{R}_i(\tau_2^{(i)})$ =0.8: $\tau_2^{(1)}$ =20.05, $\tau_2^{(2)}$ =19.08, $\tau_2^{(3)}$ =19.89, $\tau_2^{(4)}$ =19.81, $\tau_2^{(5)}$ =20.22, $\tau_2^{(6)}$ =18.47, $\tau_2^{(7)}$ =17.18, $\tau_2^{(8)}$ =20.20, $\tau_2^{(9)}$ =17.79, $\tau_2^{(10)}$ =18.39. Then the next inspection time $\tau_2 = \frac{1}{10}\sum_{i=1}^{10}\tau_2^{(i)}$ =19.11.

Step 2: An inspection and maintenance are carried out after $\Delta\tau_2$ =9.11, and the data is shown in fourth and fifth columns of Tabel 1. We then obtain M_2. In a similar way, we get that $(\hat{\eta}^{(2)},\hat{\sigma}^{(2)})$ = (5.65,4.48), and $\Delta\tau_3$ =12.82.

Step 3: Another inspection is carried out after $\Delta\tau_3$ =12.82. The data is shown in the sixth and seventh columns of Tabel 1. We also obtain M_3, and get $(\hat{\eta}^{(3)},\hat{\sigma}^{(3)})$ =(5.52,3.85), and $\Delta\tau_4$ =11.76.

Step 4: We then get M_4, $(\hat{\eta}^{(4)},\hat{\sigma}^{(4)})$ =(5.30,3.95), and $\Delta\tau_5$ =14.10, and we can continue with this procedure. As the degradation information increases, the estimator $(\hat{\eta},\hat{\sigma})$ of (η,σ) becomes more and more accurate. Then, we get $\Delta\tau_6$ =11.95, $\Delta\tau_7$ =14.72, $\Delta\tau_8$ =12.16.

For the cases of D_a =60, 70, 80, we get Table 2 to Table 4 by using the similar procedure.

Table 2: Data from inspections and maintenances and the sequential inspection time (D_a =60)

No.	1	2	3	4	5	6	7	8
$\Delta\tau_k$	10	8.69	13.23	14.55	13.86	15.33	14.29	14.15
$\hat{\eta}^{(k)}$	5.19	4.91	5.02	4.88	4.91	4.93	4.90	4.92
$\hat{\sigma}^{(k)}$	2.37	3.15	3.50	3.27	3.05	3.17	3.16	3.11
N_1	1	7	11	17	21	26	30	34
N_2	0	4	10	14	20	25	30	35

Table 3: Data from inspections and maintenances and the sequential inspection time (D_a =70)

No.	1	2	3	4	5	6	7	8
$\Delta\tau_k$	10	8.18	11.38	16.29	11.62	13.95	13.06	14.00
$\hat{\eta}^{(k)}$	5.25	5.13	5.05	5.03	4.93	4.95	4.97	4.95
$\hat{\sigma}^{(k)}$	1.78	1.81	2.17	2.34	2.54	2.57	2.75	2.77
N_1	0	8	8	13	13	17	18	22
N_2	0	2	10	13	19	23	29	33

Table 4: Data from inspections and maintenances and the sequential inspection time (D_a =80)

No.	1	2	3	4	5	6	7	8
$\Delta\tau_k$	10	8.39	12.18	12.72	10.82	13.95	10.99	14.01
$\hat{\eta}^{(k)}$	5.06	5.16	4.92	4.99	4.97	4.93	4.96	4.96
$\hat{\sigma}^{(k)}$	2.50	2.67	3.19	2.86	2.90	2.87	2.80	2.84
N_1	0	3	3	4	4	5	5	5
N_2	0	5	10	14	20	24	30	34

From above four tables, we can obtain some useful results, such as the mean inspection time interval, denoted by $\Delta\tau_{mean}$, the mean preventive maintenance times per unit operation time N_1 , and the mean replacement times per unit operation time N_2 . For different alarm threshold, $\Delta\tau_{mean}$, N_1 and N_2 are calculated as in the *Table 5.*

Table5: Different $\Delta\tau_{mean}$ *,* N_1 *and* N_2 *with respected to different* D_a

D_a	$\Delta\tau_{mean}$	N_1	N_2
50	12.08	0.38	0.38
60	13.01	0.33	0.34
70	12.31	0.22	0.34
80	11.63	0.05	0.37

From Table 5, we know that alarm threshold have little effect on the mean inspection time interval and the mean replacement times per unit operation time. However, the mean preventive maintenance times per unit operation time is very sensitive to alarm threshold, especially as alarm threshold is large and close to failure threshold, there is little preventive maintenance. It is very obvious in Table 4.

5. Conclusions and discussions

This paper extends the sequential inspection procedure in Cui et. al. (2004) to the case of multiple systems with Wiener degradation. Besides replacement, we consider preventive maintenance based on alarm threshold. We develop a sequential inspection and maintenance plan to ensure that the availability is at the required level on the view of average. We analyzed the effect of different alarm thresholds on the optimal inspection and maintenance plan. Results show that the alarm threshold has little effect on the mean inspection time interval and the mean replacement times per unit time. However, as the alarm threshold is large and close to the failure threshold, the mean preventive maintenance times per unit time decreases very obviously.

Although the procedure proposed in this paper can be implemented easily, there are several other situations that could be considered. Besides Wiener process, other degradation processes such as the gamma process and compound Poisson process can be investigated by using of the similar procedure proposed in this paper. We use the average availability criterion to determine the sequential inspection time, while other criteria can also be studied.. When the system undergoes more than one time preventive maintenance, the modelling of the system degradation is generally difficult as specific models describing the effect of successive preventive maintenance on the system degradation will be needed.

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