

# Fault Diagnosis of Hybrid Systems Using Particle Filter Based Hybrid Estimation Algorithm

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Hybrid systems are composed of discrete event dynamic systems and continuous time dynamic systems, which interact with each other. The fault diagnosis of hybrid systems is particularly difficult. To solve this problem, a hybrid estimation based hybrid systems fault diagnosis method is proposed in this paper. Based on the stochastic hybrid automaton, the operating states and fault states of the system are modelled to support the fault diagnosis of hybrid systems. Then the common particle filter based hybrid estimation algorithm is developed so as to be used in hybrid states estimation and fault diagnosis of hybrid systems, which contain controlled migration, autonomous migration and stochastic migration. To demonstrate the proposed method, a simulation experiment is employed to conduct the fault diagnosis on a typical hybrid system. And the results indicate that this method is able to diagnose the failure accurately.

## 1. Introduction

Hybrid systems contain both of the continuous real-time dynamic systems and the discrete event systems. These continuous and discrete dynamic behaviors not only co-exist but interact with each other (Schaft and Sfumacher, 2000). In practice, most dynamic systems are hybrid systems, such as the automobile, the computer, the aircraft, etc. The interaction between discrete and continuous dynamics, which makes the system more complicated, presents new challenges to fault diagnosis. Fault diagnosis of hybrid systems has become a hotspot in the control field and the computer field.

In recent years, some fault diagnosis methods are presented for hybrid systems, which can be divided into two ways. One type of methods aims at the system which self has hybrid characteristics. Proper system model is established for state monitoring of hybrid systems. That is, the state changes of hybrid systems is estimated and tracked based on the model and the observed value. Then the possible fault is judged according to the model and the estimation results. Basseville (1998) has adopted stochastic petri net to model in order to realize fault diagnosis and separation. Koutsoukos (2001) has proposed to use the hybrid automata to describe the hybrid system, so as to realize fault diagnosis of the embedded system. For hybrid systems with unknown modes, Yu (2010) has presented a method that using the hybrid differential evolution algorithm for fault location. Zhou and Liu (2010) put forward a strong state tracking particle filter (SST-PF) algorithm for the diagnosis of unknown fault of hybrid systems.

Another type of methods is for general continuous dynamic system. The discrete dynamic is used to fault modelling, in the sense that faults are regarded as a special kind of discrete event in the system. Then the system can be seen as a stochastic hybrid system with multiple discrete states. Faults are analysed through estimating discrete states of the system. Hyfbaur (2002) proposed to use the probability hybrid automata for description and Kalman filter, so as to realize the mode estimation and fault diagnosis. Mo and Xiao (2003) have presented to describe discrete dynamic by Markov chain, and modelled fault as part of Markov chain. The particle filter was used for state estimation of hybrid systems, so as to realize fault diagnosis. Zhang (1998) has put forward a fault diagnosis method of hybrid systems based on the multi-model estimate. Discrete mode of system are described as hidden Markov chain in this method. Fault detection and diagnosis is to determine the current discrete mode and continuous state according to the observed value obtained.

The existing fault diagnosis methods usually ignore the interaction between continuous dynamic and discrete dynamic, i.e. neglecting the spontaneous change of discrete state. Either the discrete mode is required to be measured directly, or the discrete dynamic is described by Markov chain (Wang et al, 2006). Generally, continuous state of hybrid systems depends on the discrete state. While discrete state relate to not only the system operation mode, but also the continuous state changes. The existing hybrid estimation method is for general continuous system, only considering stochastic migration (fault event). In this paper, the method is presented to tackle with the system which has hybrid characteristics itself. For instance, the controlled migration and the autonomous migration of the hybrid system are considered in this method. And fault diagnosis is realized by particle filter based hybrid estimation algorithm.

This paper is organized as follows. Section 2 provides the fault diagnosis model based on stochastic hybrid automaton, and defines the problem studied in the paper. In section 3, considering the interaction between discrete state and continuous state, the particle filter based hybrid estimation and diagnosis algorithm is proposed. Applying the proposed method to the fault diagnosis of the temperature control system, simulation results are presented in section 4.

## 2. The fault diagnosis model based on SHA

Hybrid Dynamical Systems (HDS), also named Hybrid Systems, is constituted by Continuous Variable Dynamical Systems (CVDS) and Discrete Event Dynamical Systems (DEDS). Among the descriptions of hybrid systems, hybrid automata is the most common model. In the hybrid automata, the discrete part of system is described as a limited directed graph (automaton). And corresponding to each discrete state, the dynamic characteristics of continuous part is expressed by a set of differential equation. This kind of hybrid automata is commonly used to describe the deterministic events. And it is required to improve to describe the discrete event of fault. Considering the stochastic fault event and making each fault state as a discrete state, the model which contains both of the normal operating mode and faulty mode is established. Then the stochastic hybrid automaton (SHA) is built.

The SHA can be presented by  $\langle S, X, Y, T, F, H \rangle$ , where finite set  $S$  is the discrete mode space,  $X$  is the continuous state space,  $Y$  is the observation space,  $T$  is the transition function between modes,  $F$  is the set of state functions and  $H$  is the set of observation functions. The SHA contains various kinds of migration in the system and the continuous dynamic characteristics of each mode. The possible mode set can be obtained according to the model and the current mode. The discrete changes of hybrid dynamic system model contain controlled migration, autonomous migration and stochastic migration. Controlled migration is the transformation caused by a control instruction; autonomous migration is the automatic transformation caused by the changing of continuous variables; both of them belong to switching caused by the model itself. Stochastic migration is the unexpected behaviour which causes component failures of system (fault). And it belongs to the external behaviour.

Considering three kinds of migration, both the operating state and the fault state of the system are modelled in this paper. Fault state is considered in the discrete mode  $S$ , and stochastic migration is carried out according to the failure rate in mode transition  $T$ . Then the fault diagnosis model based on SHA is obtained. The schematic diagram is shown in Figure 1.

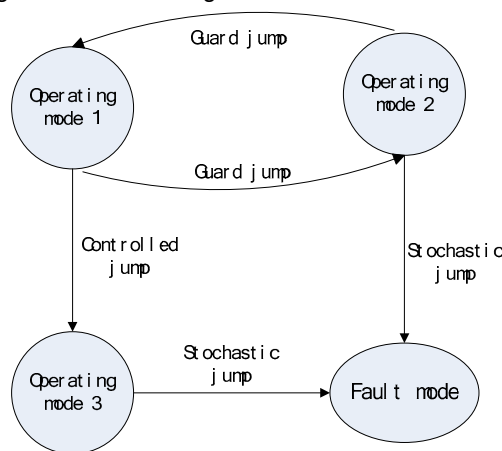


Figure 1 the schematic diagram of SHA

System mode usually described by the discrete state. If the discrete state is decided, the dynamic characteristics of hybrid systems would be determined. When a system is in a mode, say  $s_t$ , its behaviour is given by the following state-space model:

$$\begin{aligned} x_{t+1} &= f_{s_t}(x_t, u_t) + v_t \\ y_t &= h_{s_t}(x_t) + n_t \end{aligned} \quad (1)$$

where  $x_t$  is the continuous state vector at time  $t$ ,  $x_0$  is the initial condition;  $s_t$  is the discrete state at time  $t$ , the two states are called hybrid state collectively, described by  $(s_t, x_t)$ ;  $y_t$  is the observation vector at time  $t$ ,  $u_t$  is the input vector of system,  $f_{s_t}$  is the state function vector in mode  $s_t$ ,  $h_{s_t}$  is the observation function vector in mode  $s_t$ ,  $v_t$  and  $n_t$  are independent, zero-mean-value random process noise and measure noise with Gaussian distributed.

After the construction of the SHA model, fault diagnosis is converted into discrete state estimation problem. According to the estimate for discrete state we can judge whether the system fault, and which faulty mode are system in. So this kind of problems of fault diagnosis for hybrid systems can be described as: given the hybrid system model, utilize the observation sequence  $Y_t = (y_0, y_1, \dots, y_t)$  and the control sequence  $U_t = (u_0, u_1, \dots, u_t)$  to get the discrete state  $S_t = (s_0, s_1, \dots, s_t)$ . However, the continuous state and discrete state interact in hybrid systems. The estimation of discrete state will depend on the continuous state, on the other hand, the estimation of continuous state need to get discrete state information. Due to the dynamic interaction between two kinds of information, therefore, hybrid estimation must be taken advantage of to judge the state of system.

### 3. The particle filter based hybrid estimation and diagnosis algorithm

Based on SHA model, the particle filter algorithm is used to conduct the state estimation and diagnosis in this paper. Particle filter is a kind of statistical filtering method based on Monte Carlo method and Bayesian estimation (Tafazoli et al, 2006). The distribution probability of state is approximated by the finite-sample set. Given the observation value, the sample set is corrected according to the level of similarity of state. The process is repeated to calculate the state posterior distribution, and use some analytical structure of model to simplify the algorithm. The estimates are similar to real posterior probability of state variables when the sample size is large enough. However, when apply the particle filter algorithm to hybrid estimation, the current references only aim at ordinary continuous systems with stochastic migration.

The hybrid estimation algorithm based on particle filter is proposed in this paper, which supports several migrations of hybrid systems. First, get an initial particle set of the discrete state variables and continuous state variables according to the prior distribution. Second, sample according to the mode transfer function and use the differential equation of each mode to get a hybrid state set, in which each sample has the same weight. Then evaluate the sample according to the measured value, and get the proportionate weighted value and normalized the sample set. Then resample according to the weights. That is, increase or decrease the corresponding particles according to the similarity degree with measurement, and get a new equal weighted sample set as the initial sample set of next cycle. Repeat the process, the approximate distribution of the set will gradually close to the real distribution. The mode which has maximum number in the sample set should be considered as mode estimation of system. And continuous state variables can be obtained similarly, realizing the hybrid estimation of hybrid systems.

The specific hybrid estimation and diagnosis algorithm is shown as follows.

1) Initialization

For  $i = 1, 2, \dots, N$ , sample the equal weighted particle set  $\{s_0^i, x_0^i, 1/N\}_{i=1}^N$  from prior probability  $p(s_0)$  and  $p(x_0)$ .  $N$  is the number of particles.

2) Prediction

Compute  $\{\tilde{s}_{t|t-1}^i, \tilde{x}_{t|t-1}^i\}_{i=1}^N$ . Compute discrete state  $\{s_{t|t-1}^i\}_{i=1}^N$  with Mode transfer function  $T(s_t | s_{t-1})$  and compute the corresponding continuous state  $\{\tilde{x}_{t|t-1}^i\}_{i=1}^N$  with the differential equations of system

$f_{s_t^i}(x_{t-1}^i, u_{t-1})$ . There are three kinds mode transfer. One is to receive control instruction from outside; one is to transfer automatically when continuous states of system reach a certain conditions; the other one is the stochastic event (fault) which is described by probability.

### 3) Computation of important weight

Compare observations with prediction of each particle to compute the important weight  $\omega_t^i = p(y_t | \tilde{s}_{t|t-1}^i, \tilde{x}_{t|t-1}^i)$ , and get the particle set  $\{\tilde{s}_{t|t-1}^i, \tilde{x}_{t|t-1}^i, \omega_{t|t-1}^i\}_{i=1}^N$ . Then normalize the weights

$$\tilde{\omega}_{t|t-1}^i = \omega_{t|t-1}^i / \sum_{i=1}^N \omega_{t|t-1}^i.$$

### 4) Resampling

Resample  $N$  new equal weighted particles  $\{s_t^i, x_t^i, 1/N\}_{i=1}^N$  with replacement from particle set  $\{\tilde{s}_{t|t-1}^i, \tilde{x}_{t|t-1}^i\}_{i=1}^N$  according to the important weights  $\tilde{\omega}_{t|t-1}^i$ . Through resampling we can take out the particles with smaller weights, copy particles with larger weights, and improve the efficiency of Monte Carlo method.

### 5) Estimation

The estimation of mode is described by the most likely particle in each step, and the continuous state is computed by the most likely mode.

$$\hat{s}_t = \arg \max_j \sum_{i \in \hat{Q}_j} \omega_t^i \quad (2)$$

$$\hat{x}_t = \sum_{i \in \hat{Q}} \omega_t^i x_t^i / \sum_{i \in \hat{Q}} \omega_t^i \quad (3)$$

where  $\hat{Q}_j = \{i | s_t^i = j\}$ , and  $\hat{Q} = \{i | s_t^i = \hat{s}_t\}$ .

### 6) Fault diagnosis

According to the estimation of discrete state it is able to judge whether the system fault, and which faulty mode system in. Set  $t = t + 1$ , and back to step 2.

When the number of particles approaches to infinite, the particle filter can guarantee that the estimates converge to the true values. Theoretically, therefore, the hybrid estimation method can converge to the true hybrid state when the number of samples approaches infinity.

## 4. Simulation experiments

The following temperature system is used as an example to discuss the algorithm. The sensor fault is diagnosed by the particle filter based hybrid estimate algorithm. The state space equations of the system are:

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t \\ y_t &= Cx_t \end{aligned} \quad (4)$$

$$\text{Where } A = \begin{bmatrix} 0.9934 & 0 & 0 & 0 \\ 0.0066 & 0.9934 & 0 & 0 \\ 0 & 0.005 & 0.9995 & 0 \\ 0 & 0 & 0.005 & 0.9995 \end{bmatrix}, B = \begin{bmatrix} 0.053156 \\ 0.00018 \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

This dynamic is regarded as the normal mode of system. When  $u_t \neq 0$ , the dynamic is identified as mode 1; when  $u_t = 0$ , the dynamic is identified as mode 2.

The faulty mode is given by (5) where noises has been added to the normal system model.

$$\begin{aligned} x_{t+1} &= \mathbf{A}x_t + \mathbf{B}u_t + v_t \\ y_t &= \mathbf{C}x_t + G_t \delta_t + n_t \end{aligned} \quad (5)$$

where  $v_t$  and  $n_t$  are zero mean Gaussian random variables with variances 0.05. They are mutually independent.  $\delta_t$  is the fault function and  $\delta_t = \begin{bmatrix} 10 \\ 100 \end{bmatrix}$ .  $G_t$  is the diagonal matrix and presents the timescales of fault. When  $G_t = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ , fault 1 occurs. When  $G_t = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ , fault 2 occurs.

The schematic diagram of SHA model of system is shown in Figure 2.  $s=1$  and  $s=2$  present two normal operating modes of system and can switch with each other. The remainder four states are faulty states.  $s=3$  presents the state when fault 1 has occurred in mode 1.  $s=4$  presents the state when fault 1 has occurred in mode 2. And so on. Assume that the probability of two faults are both 0.002, and the system mode is irreversible after fault occurs.

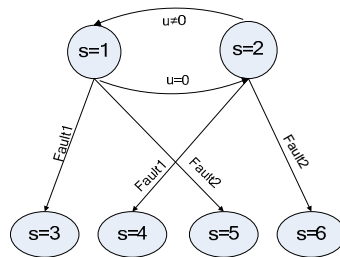


Figure 2 the schematic diagram of SHA model of system

The setting of the simulations are: for  $1 \leq t < 600$ , the system is in mode 1; for  $600 \leq t < 800$ , the system is in mode 2; and for  $800 \leq t < 1200$ , the system is in mode 4. Let the initial mode=1 and the initial state  $x_0 = [1 \ 1 \ 1 \ 1]^T$ . The number of particles is set to 1000. The results of simulation are shown in Figure 3 and Figure 5.

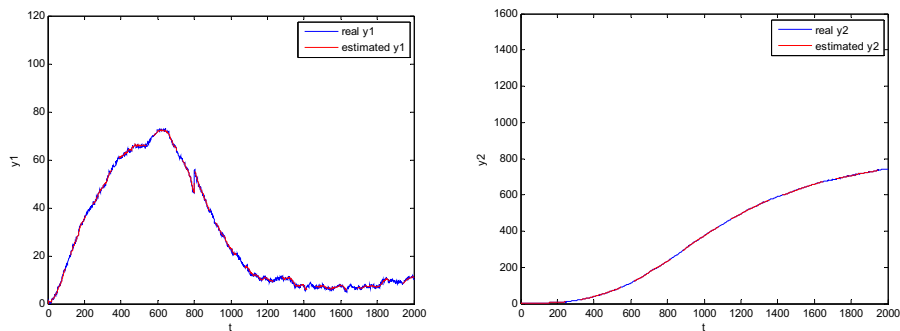


Figure 3 the estimation of observations

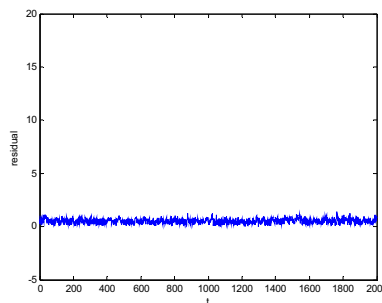


Figure 4 the residual

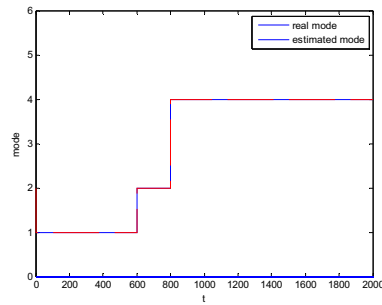


Figure 5 the estimation of mode

Figure 3 is the estimation of observations and Figure 3 is the estimation of mode. We can find that the particle filter tracks the modes very well. As  $t = 600$ , a control event occurs and then the system turn into mode 2. As  $t = 800$ , a fault event occurs and the system turns into mode 4.

The simulation experiment indicates that the particle filter based hybrid estimation algorithm can monitor the state of system and diagnose the faults in real time. But when the number of modes increases, the tracking of the system will be a bit slower. And when the variance of the noise increases, the algorithm may performs not very well.

## 5. Conclusions

In this paper, the SHA model is used to describe the hybrid system that faults may happen potentially. Fault diagnosis is converted into a discrete state estimation problem of the hybrid system. The particle filter algorithm is used to estimate the hybrid state of system. Based on this, fault diagnosis is able to apply to the hybrid system. The simulation results shows that the proposed method is effective, and it can be used to deal with the practical fault diagnosis problem of hybrid systems. However, the method is based on the standard particle filters, which may not do well in other complex systems. Besides, the number of modes and the variance of the noise also influence the algorithm performance and efficiency.

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