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An Intelligent Prognostic Method for SSADT Based on SVM

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The support vector machine (SVM), which has long-term prediction period, strong generalization ability and high prediction accuracy, provides an efficient new way for life prediction of accelerated degradation testing (ADT). In this paper, an intelligent prognostic model for step-stress ADT (SSADT) based on SVM is proposed. The SSADT data of superluminescent diode (SLD) is utilized to validate the proposed method.

1. Introduction

Along with the continuous improvement of modern product design, manufacturing level and material, product reliability becomes increasingly higher and life becomes longer. Traditional reliability analysis method based on binary failure data (normal and fail) is not applicable now. It is a continuous performance degradation process from the product initial state to the final failure. Since the failure mechanism of product can be traced to its potential performance degradation process, the thought of utilizing performance degradation data of product to predict its lifetime and reliability is proposed. Accelerated Degradation Testing (ADT) is the process of testing a product by subjecting it to conditions in excess of its normal usage in an effort to collect more degradation data in a short time period to extrapolate and predict lifetime and reliability of products under normal conditions.

There are two main ADT methods presently: constant-stress ADT (CSADT) and step-stress ADT (SSADT). CSADT divides the samples into several groups. Each group is tested under a certain accelerated stress level. In SSADT, all specimens are tested under the accelerated stress increasing step by step. The implementation and data analysis method of CSADT is relatively simple, and many research achievements have been obtained (Nelson, 1990; Meeker et al., 1998; Deng et al., 2007). By contrast, the SSADT is more complex, and the achievements are relatively less (TSENG and WEN, 2000; Li et al., 2008; Yuan et al., 2012). However, because of the less sample size and higher testing efficiency, the SSADT attracts more and more attention.

In this paper, Statistical Learning Theory is introduced to the statistical analysis of SSADT, and an intelligent prognostic model for SSADT based on support vector machine (SVM) is proposed. The method overcomes the difficulty brought by the small sample data in a certain extent and further enriches the theory of ADT evaluation.

2. Theory of Support Vector Machine for regression

Support vector machine (SVM) was developed by V. N. Vapnik (Vapnik, 1995), which is a novel learning machine based on Statistical Learning Theory (SLT). SVM minimizes the true risk by the principle of Structure Risk Minimization (SRM). The basic idea of SVM for regression is to map the input data into a high-dimensional feature space by a nonlinear mapping Φ and to perform linear regression in this space. The theory of SVM for regression is briefly introduced as follows (Xu et al., 2007).

Given a set of training data $\{(\boldsymbol{x}_i, y_i), \ldots, (\boldsymbol{x}_l, y_l)\}$, $i=1, 2, \ldots, l$, where $\boldsymbol{x}_i \in \mathbf{R}^n$ denote input patterns, $y_i \in \mathbf{R}$ are the targets and l is the total number of training samples. In SVM for regression, the goal is to find a function $f(x)$, i.e. an optimal hyperplane, which has at most ε deviation from the actually obtained targets y_i for all the training data and is as flat as possible. The form of functions is denoted as

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$$y = f(x) = \langle \omega, \Phi(x) \rangle + b \quad \text{with} \quad \Phi : R^n \to F, \omega \in F \tag{1}$$

where $\Phi(\cdot)$ is a nonlinear mapping by which the input data x is mapped into a high dimensional space $\boldsymbol{F}$, and $\langle \cdot,\cdot \rangle$ denotes the dot product in space $\boldsymbol{F}$. The unknown variables ω and b are estimated by minimizing the regularized risk function $R_{reg}[f]$

$$R_{\text{reg}}[f] = \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{l}\zeta(f(x_i) - y_i) \tag{2}$$

$$\zeta(f(x) - y) = \max\{0, |y - f(x)| - \varepsilon\} \tag{3}$$

where C is a constant determining the trade-off between the flatness of the regularized term ($\|\omega\|^2/2$) and the empirical error (the second term). $\zeta(\cdot)$ is the ε-insensitive loss function, and e is the tube size.

By introducing the non-negative slack variables $\xi_i^{(*)}$, Eq. 2 is transformed into the following convex constrained optimization problem:

$$\begin{aligned} &\text{Minimize} \quad \Gamma(\omega, \xi, \xi^*) = \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{l}\zeta(\xi_i + \xi_i^*), \\ &\text{Subject to} \quad \langle \omega, \Phi(x_i) \rangle + b - y_i \le \varepsilon + \xi_i \\ &\qquad\qquad\quad y_i - \langle \omega, \Phi(x_i) \rangle - b \le \varepsilon + \xi_i^* \\ &\qquad\qquad\quad \xi_i, \xi_i^* \ge 0, \quad i = 1, 2, \cdots, l. \end{aligned} \tag{4}$$

According to Wolfe's Dual Theorem and the saddle-point condition, the dual optimization problem of the Equation 4 is obtained as the following form:

$$\begin{aligned} &\text{Minimize} \quad W(\alpha_i^{(*)}) = \frac{1}{2}\sum_{i,j=1}^{l}(\alpha_i^* - \alpha_i)(\alpha_j^* - \alpha_j)\langle \Phi(x_i), \Phi(x_j) \rangle + \varepsilon\sum_{i=1}^{l}(\alpha_i^* + \alpha_i) - \sum_{i=1}^{l} y_i(\alpha_i^* - \alpha_i) \\ &\text{Subject to} \quad \sum_{i=1}^{l}(\alpha_i - \alpha_i^*) = 0 \\ &\qquad\qquad\quad 0 \le \alpha_i^{(*)} \le C, \quad i = 1, 2, \cdots, l. \\ &\text{With} \quad \omega = \sum_{i=1}^{l}(\alpha_i^* - \alpha_i)\Phi(x_i) \end{aligned} \tag{5}$$

where $\alpha_i^{(*)}$ are the nonnegative Lagrange multipliers that can be obtained by solving the convex quadratic programming problem stated above. Finally, by exploiting the Karush-Kuhn-Tucker (KKT) conditions, the decision function given by Equation 1 gets the following form

$$f(x) = \sum_{i=1}^{l}(\alpha_i^* - \alpha_i)\langle \Phi(x_i), \Phi(x) \rangle + b \tag{6}$$

In order to get the dot product in the feature space simply and avoid the curse of dimensionality, the kernel function $k(x_i, x_j) = \langle \Phi(x_i), \Phi(x_j) \rangle$ is introduced, which satisfies the Mercer condition. The most commonly used kernel functions are:

(a) The polynomial kernel $k(x, x') = (\langle x, x' \rangle + c)^p, \quad p \in N, c \ge 0$

(b) The Radial Basis Function (RBF) kernel $k(x, x') = \exp\left(-\|x - x'\|^2 / (2\sigma^2)\right)$

(c) The Sigmoid kernel $k(x, x') = \tanh(b\langle x, x' \rangle + \theta)$

3. Step-stress Accelerated Degradation Testing

3.1 Stress Applied Method

A group of accelerated stress level is selected before the SSADT, i.e. $S_1<S_2<\ldots<S_K$, which are all harsher than normal stress level S_0. At the beginning, all the specimens are tested under stress level S_1. After a predetermined duration time, stress level is changed to S_2. So continue, the testing is conducted until it reaches the censored time or all specimens' performance parameters degrade to a certain threshold while no failure happening.

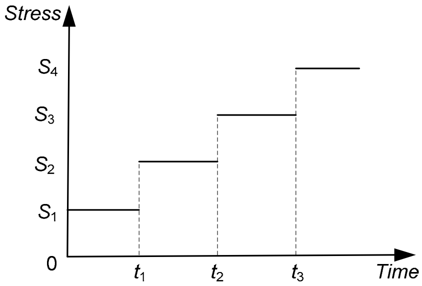

Figure 1: The process of stress applied in SSADT

SSADT has the advantage of fewer specimens, which is an efficient way to solve the problem of the small sample. Since the same products are tested under all stress levels in SSADT, cumulative damage should be considered when making SSADT data analysis. Therefore, it is necessary to convert the test data under each accelerated stress level into the normal stress level.

3.2 Basic Hypothesis

Data analysis of SSADT is basically under the following hypothesis (Li et al., 2008):

1) Product failure mechanism and failure mode do not change under each accelerated stress level;

2) Product has a regular performance degradation process and the process is irreversible;

3) A cumulative exposure model holds. That is, the remaining life of specimens depends only on the current cumulative damage fraction it has happened and current stress - regardless how the fraction accumulated (Nelson, 1990).

According to Hypothesis 3, the testing data obtained by SSADT can be converted into the same stress level. Li et al. (2008) has provided the specific conversion algorithm.

4. The Intelligent Prognostic Method for SSADT Based on SVM

4.1 The SVM Prognostic Model

Product performance degradation data can be regarded as a time series which reflects the product state. According to Kolmogrov Theorem, any time series can be regarded as an input-output system which is determined by nonlinear mechanism. The idea of SVM prognostic model is finding the mapping f on the basis of the history data to approach the hidden nonlinear mechanism F in time series. The mathematical description of SVM prognostic model is as follows.

Given a time series $X=\{x_1,x_2,\cdots,x_k\}$, the question is how to predict x_k while $\{x_{k-m},x_{k-m+1},\cdots,x_{k-1}\}$ are known. Solving this problem needs to establish the mapping f: $R^m \rightarrow R$, where $\{x_{k-m},x_{k-m+1},\cdots,x_{k-1}\}\in R^m$, $x_k\in R$. So we can get

$$x_k = f(x_{k-m},x_{k-m+1},\cdots,x_{k-2},x_{k-1}) \tag{7}$$

For the modeling of time series $X=\{x_1,x_2,\cdots,x_k\}$, in order to establish the mapping relationship between the moving time window $X_k=\{x_{k-m},x_{k-m+1},\cdots,x_{k-2},x_{k-1}\}$ and the output $\{x_k\}$, phase space reconstruction should be performed firstly.

$$X_{re}=\begin{bmatrix}\vec{x}_1\\ \vec{x}_2\\ \vdots\\ \vec{x}_{k-m}\end{bmatrix}=\begin{bmatrix}x_1 & x_2 & \cdots & x_m\\ x_2 & x_3 & \cdots & x_{m+1}\\ \vdots & \vdots & \ddots & \vdots\\ x_{k-m} & x_{k-m+1} & \cdots & x_{k-1}\end{bmatrix},\quad Y_{re}=\begin{bmatrix}x_{m+1}\\ x_{m+2}\\ \vdots\\ x_k\end{bmatrix} \tag{8}$$

where, X_{re} is the input matrix after reconstruction, Y_{re} is the corresponding vector. m is the prediction embedded order. The principle of Final Prediction Error (FPF), which can identify AR model order in time series analysis, is utilized to obtain the best prediction order *m* (Guo, 2009).

According to the decision function (Eq. 6) of SVM for regression, SVM regression model is obtained as following form

$$x_t = \sum_{i=1}^{k-m} (\alpha_i^* - \alpha_i) K(\vec{x}_i, \vec{x}_{t-m}) + b \tag{9}$$

where $t = m+1,\ldots,k$. Then, one-step SVM prediction model is

$$\hat{x}_{k+1} = \sum_{i=1}^{k-m} (\alpha_i^* - \alpha_i) K(\vec{x}_i, \vec{x}_{k-m+1}) + b \tag{10}$$

where $\hat{x}_{k+1}$ denotes the $k+1^{\text{th}}$ prognostic value of the raw data, $\vec{x}_{k-m+1} = \{x_{k-m+1}, x_{k-m+2}, \cdots, x_k\}$ denotes k-$m+1^{\text{th}}$ row of matrix X_{re}, that is the sets of m elements before x_{k+1} in X.

$\hat{x}_{k+1}$ could be used to construct new vector $\vec{x}_{k-m+2} = \{x_{k-m+2}, x_{k-m+3}, \cdots, x_k, \hat{x}_{k+1}\}$, and the $k+2^{\text{th}}$ recursive prognostic value can be obtained by making it as the input. Further generalization can obtain the *n*-step recursive prognostic model as follow.

$$\hat{x}_{k+n} = \sum_{i=1}^{k-m} (\alpha_i^* - \alpha_i) K(\vec{x}_i, \vec{x}_{k-m+n}) + b \tag{11}$$

where $\hat{x}_{k+n}$ denotes n^{th} step recursive prognostic value, $\vec{x}_{k-m+n} = \{x_{k-m+n}, x_{k-m+n+1}, \cdots, x_k, \hat{x}_{k+1}, \cdots, \hat{x}_{k+n-1}\}$ denotes k-m+n^{th} row of matrix X_{re}.

4.2 The Proposed Intelligent Prognostic Method

The SVM intelligent method is utilized to analyze the testing data of SSADT and make prognostic in this paper. The research program is shown in Figure 2.

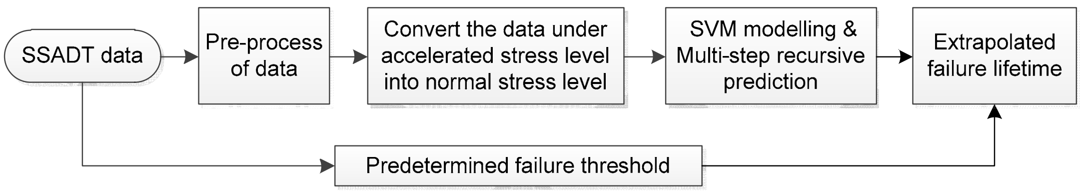

Figure 2: Flow chart of prognostic method for SSADT based on SVM

Generally, the performance degradation data obtained by SSADT usually have some abnormal values, irrelevant trending fluctuations or temperature drift phenomenon. So, the pre-process of the testing data should be conducted firstly.

Then, the degradation data under accelerated stress level needs be converted into the operation stress level based on the hypothesis of cumulative damage (Li et al., 2008).

Finally, multi-step recursive prediction for the converted degradation data under normal stress level should be conducted by utilizing the proposed prognostic model for performance degradation based on SVM. According to the predetermined failure threshold, extrapolated failure lifetime of each specimen can be obtained. Then, the corresponding reliability curve can also be obtained by fitting the distribution of extrapolated failure lifetime.

5. Case study

Integrating the merits of LD's big output power and LED's broad spectrum width, superluminescent diode (SLD) is the most important and fragile part of fiber optic gyroscopes (FOGs) (Chao et al., 2012).

For evaluating the lifetime and reliability of SLD, a temperature SSADT with 4 stress levels has been conducted. The sample size is 3 and the monitoring parameter is the output optical power of SLD. The parameters of the SSADT are in Table 1 and the SLDs' performance degradation raw data obtained by SSADT are shown in Figure 3(a).

Table 1: The test parameters of SSADT

Stress level	S_1	S_2	S_3	S_4
Temperature/℃	60	80	100	110
Duration time/hours	2420	716	301	342

Since the performance parameters of SLD have the characteristic of temperature drift, the degradation data have step changes. The pre-process of the testing data is performed firstly in order to exclude the effects of temperature drift. The pre-process results are shown in Figure 3(b).

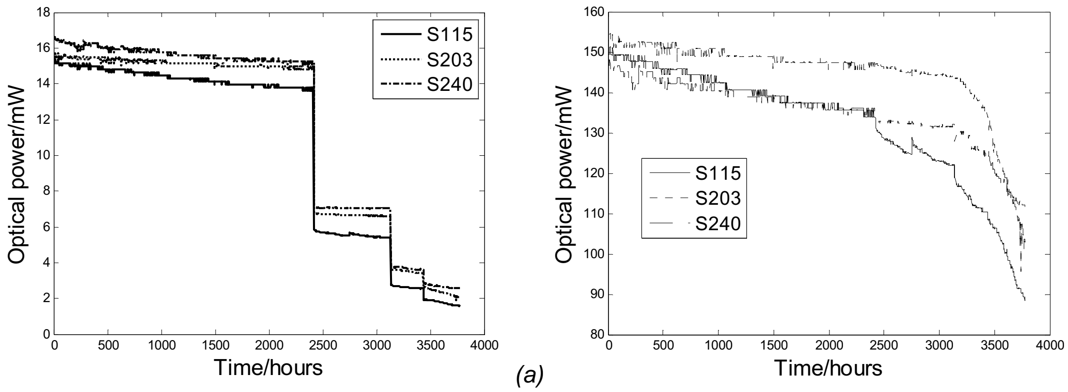

Figure 3: SLDs' degradation data obtained by SSADT: (a) raw data, (b) data after pre-process

Then, the performance degradation rates are obtained by regression fitting the testing data under each stress level. Because the sensitive stress type of SLD is temperature, the Arrhenius model is considered to acceleration model, i.e., the relationship between the SLD performance degradation rate and temperature stress level is $d(T) = \exp(A - B/T)$. After obtaining the acceleration model parameters of each specimen through least square fitting, the degradation data of each specimen under accelerated stress can be converted into normal stress level 25℃, as shown in Figure 4.

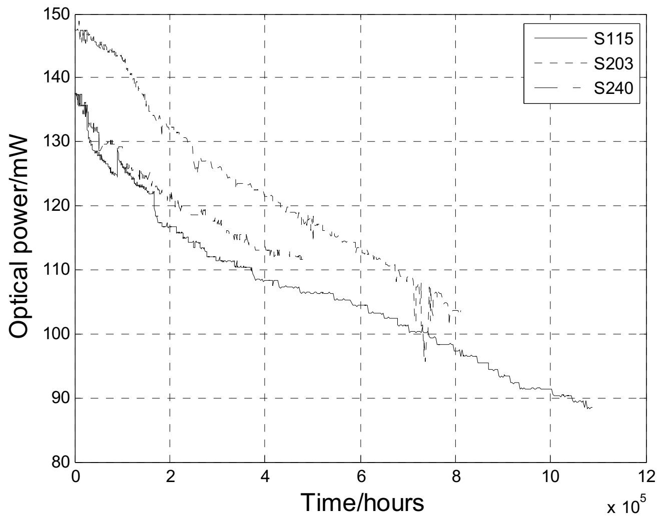

Figure 4: the degradation data converted into normal stress level

Finally, Support vector machine modeling and multi-step recursive prediction for each SLD are performed, and the prognostic results of extrapolated failure lifetime for 3 specimens are shown in Table 2. Where, the failure threshold value is 50% of the initial value under normal stress level. The reliability curve obtained by Weibull distribution fitting is shown in Figure 5.

Table 2: extrapolated failure life of each sample under normal stress level

Sample number	S115	S203	S240
Extrapolated failure life /hours	3.8986×10^5	6.5558×10^5	4.9671×10^5

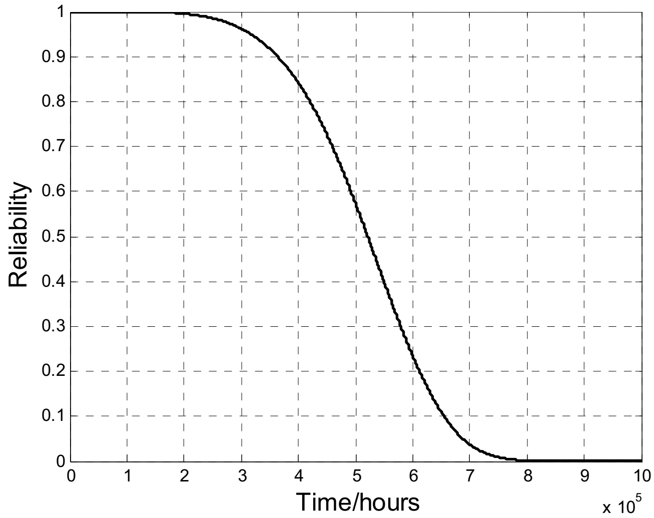

Figure 5: reliability curve of SLD under normal stress level

6. Conclusion

SVM method is introduced to analyse SSADT data and lifetime prognostic model is derived. The SSADT data of SLD is utilized to demonstrate the validity and applicability of the proposed intelligent prognostic method. The proposed method can evaluate product life and reliability in the case of zero-failure data, which has certain reference significance to the high reliability and long life products.

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