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Multi-Sensor Degradation Data Analysis

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Multiple sensors are commonly used for degradation monitoring of critical components. The degradation indicators derived from these sensors are significantly different in terms of scale and interpretation. Since these indicators reflect the same degradation path but have different scales and noise, the fusion of the data corresponding to these indicators is important for the condition monitoring and accurate reliability estimation of the components. In this paper, we develop a unified degradation path that considers statistics-based degradation and physics-based degradation models. We also estimate the reliability of such components. Simulation results show that the unified degradation model approximates the actual degradation path effectively. Additionally, more accurate estimates of reliability are obtained when compared with individual degradation paths.

1. Introduction

For a crucial component of a system, its real time degradation condition should be monitored in order to make maintenance decisions in a timely manner. As more advanced sensors are developed, it becomes more economical and feasible to utilize multiple sensors on crucial components in order to obtain more accurate reliability estimates and perform maintenance and replacement in a timely fashion. The major reason for multi-sensor degradation monitoring is that degradation information provided by a single sensor may be unreliable and probably incomplete.

Multi-sensor degradation monitoring also generates new issues and challenges, one of which is the fusion of different sources of information. The challenge lies in three aspects. First, collected from different types of sensors working based on diverse physics phenomena, the scales of different degradation indicators may vary dramatically. Thus the interpretations of these degradation indicators may differ totally from each other. Second, some sensors may be more accurate than others since they are more sensitive in certain degradation stages. Third, these indicators are usually correlated with each other since they reflect the same underlying degradation process, and hence it is necessary to interpret these indicators together.

Therefore, it is of great significance that a unified degradation path can be obtained from multiple degradation indicators. Additionally, it is important to derive reliability information metrics, i.e. reliability and MTTF, from the multiple sources of degradation information to assist maintenance decision making.

Degradation paths are usually modelled by stochastic processes and physics-based models. Brownian motion and gamma processes are widely used stochastic processes in degradation modelling. McPherson (2010) proposes three most often occurring physics-based degradation models. However, so far the modelling of multi-dimensional correlated degradation processes has seldom been considered. Moreover, the degradation modelling combining physics-based and stochastic models has not been investigated.

Even though significant research has been conducted on multi-sensor data fusion for fault diagnosis and prognosis, sparse work has addressed data fusion of multi-sensor degradation information or unified degradation path. Wei et al. (2011) weigh different sensor signals by minimizing the uncertainty of unified measurements, but the data from different sensors are considered as independent instead of correlated with each other.

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Reliability estimation for stochastic degradation models is found in many references. A recent study by Molini et al. (2011) presents the distribution for first passage time of Brownian motion with time dependent drift and diffusion coefficients. However, the research on the reliability estimate for multi-dimensional correlated degradation processes is rare. The first passage time distribution of even 2-dimensional Brownian motion with drift is quite complex (Dominé and Pieper, 1993). It is more difficult to obtain explicit reliability functions for more dimensions of more advanced degradation processes.

Therefore, we investigate the use of multi-sensor information to obtain a unified degradation path using three main steps. First, degradation indicators from multiple sensors are modelled as a k - dimensional correlated Brownian motion. Additionally, physics-based degradation model is considered in combination with stochastic process based models. Second, a unified degradation path is obtained by averaging different dimensions of degradation paths considering weights that are determined utilizing lead-lag correlation between different paths. Third, reliability information is estimated based on multi-sensor degradation information by using moving block bootstrapping method, which is verified to be more accurate than that derived from single degradation paths.

2. Multi-sensor Degradation Modeling

2.1 Multi-dimensional correlated Brownian motion based degradation modelling

For critical component, degradation indicators can be extracted from multiple sensors. These degradation indicators with different scales and interpretations are usually correlated with each other since they reflect the same underlying degradation process. Brownian motion with drift has been widely used to model single degradation path. For multiple degradation indicators, Multi-dimensional Correlated Brownian motion with drift (MCB) is used to model the degradation process under the assumption of linearly increasing degradation. MCB can be specified as (Birkbeck University of London, 2009)

$$W(t) = H[Mt + Z(t)] + w_0 \tag{1}$$

where $W(t)$ is the k - dimensional MCB, $M = [\mu_1, \mu_2, ..., \mu_k]'$ is the vector of drift coefficients, H is the Cholesky decomposition of correlation matrix P among different dimensions of $W(t)$, $Z(t)$ is a vector of k Brownian motions, i.e. $Z(t) = [\sigma_1 B_1(t), \sigma_2 B_2(t), ..., \sigma_k B_k(t)]'$ where σ_i is the diffusion coefficient of the i th Brownian motion, and $w_0 = [w_{0,1}, w_{0,2}, ..., w_{0,k}]'$ is the vector of initial values.

$W(t)$ has k dimensions, each can be described as a Brownian motion with drift.

$$W_i(t) = \lambda_i t + \theta_i B_i(t) + w_{i,0} \tag{2}$$

where $W_i(t)$ is the i th dimension of $W(t)$, λ_i and θ_i are its drift and diffusion coefficients respectively, and $B_i(t)$ is a standard Brownian motion.

From Eq. (1) and (2) it follows that $\lambda_i = \sum_{j=1}^{k} h_{ij}\mu_j$ and $\theta_i = \sqrt{\sum_{j=1}^{k} h_{ij}^2 \sigma_j^2 t}$, and

$$W_i(t) = \sum_{j=1}^{k} h_{ij}\mu_j t + \sum_{j=1}^{k} h_{ij}\sigma_j B_i(t) + w_{i,0} \tag{3}$$

2.2 Physics-based degradation model with randomness

In this paper, physics-based model is also considered to model degradation processes. The power law model is used since it is the most widely observed degradation model in practice (McPherson, 2010). Combining randomness, the power law degradation model is specified as Eq. (4)

$$Y(t) = \beta_0 t^m + \delta\sqrt{\beta_0 t^{m-1}} B(t) + Y_0 \tag{4}$$

where $Y(t)$ is the degradation process, β_0 and m is power law coefficients, Y_0 is its initial value of degradation, $\delta\sqrt{\beta_0 t^{m-1}}$ is the diffusion coefficient at time t and $B(t)$ is a standard Brownian motion.

3. Unified degradation path based on correlation information

In order to determine the degradation condition of a critical component, it is necessary to derive a unified degradation path from multi-sensor degradation information. The correlations among different degradation paths are important information, and are used to obtain weight for each path; hence the weight-averaged unified degradation path approach is utilized in this paper.

The weights are calculated by employing and modifying the leadership score proposed by Wu et al. (2010) considering the PageRank proposed by Brin and Page (1998). The leaders in a group of time series (degradation paths) behave ahead of other time series, and are considered as "good representatives" of the whole group. In this paper, the normalized leadership score is perhaps an effective approach to weigh the different degradation paths in order to obtain a unified degradation path. Suppose there are k single degradation paths, the leadership score at time t is obtained using the following four steps.

Step 1. A sliding window of length g for the i th degradation path, $D_i(t)$, is defined as a subsequence of the degradation path, which is $s_{t,g}^i = (D_{i,t-g+1},...,D_{i,t})$. The lagged correlation between two sliding windows $s_{t,g}^i$ and $s_{t,g}^j$ of two degradation paths D_i and D_j at lag l, denoted as $\rho_{t,g}^{i,j}(l)$, can be calculated by Eq. (5), see (Wu et al., 2010)

$$\rho_{t,g}^{i,j}(l) = \begin{cases} \dfrac{\sum_{\tau=t-g+1}^{t-l}(D_{i,\tau+l} - \bar{s}_{t,g-l}^i)(D_{j,\tau} - \bar{s}_{t-l,g-l}^j)}{\sigma_{t,g-l}^i \sigma_{t-l,g-l}^j}, & l \geq 0 \\ \rho_{t,g}^{j,i}(-l), & l < 0 \end{cases} \tag{5}$$

where $\bar{s}_{a,b}^c$ and $\sigma_{a,b}^c$ are the mean value and standard deviation of sliding window $s_{a,b}^c$. It should be noted that lag l should not exceed $(g/2)$ and has to satisfy lag span $|l| \leq q \leq g/2$.

Step 2. The lagged correlation values between any two degradation paths over the entire lag span are aggregated by calculating the expected correlation value conditioning on l. That is (Wu et al., 2010)

$$E_t^{ij} = \max\{E_t^{ij}(\rho \mid l \geq 0), E_t^{ij}(\rho \mid l < 0)\} \tag{6}$$

where

$E_t^{ij}(\rho \mid l \geq 0) = \sum_{l=0}^{q} \max\{\rho^{ij}(l), 0\} \Pr(l \mid l \geq 0)$ and $E_t^{ij}(\rho \mid l < 0) = \sum_{l=-q}^{-1} \max\{\rho^{ij}(l), 0\} \Pr(l \mid l < 0)$. Note that the uniformly distributed conditional probabilities $\Pr(l \mid l \geq 0) = 1/(q+1)$ and $\Pr(l \mid l < 0) = 1/q$.

Step 3. Leadership matrix L_t is determined based on the values of $E_t^{ij}(\rho \mid l \geq 0)$ and $E_t^{ij}(\rho \mid l < 0)$, the third criterion is added by authors to enhance the robustness of this method.

(1) If $E_t^{ij}(\rho \mid l < 0) > E_t^{ij}(\rho \mid l \geq 0)$, the i th degradation path leads the j th one, and $L_t^{ij} = 1$;

(2) If $E_t^{ij}(\rho \mid l < 0) < E_t^{ij}(\rho \mid l \geq 0)$, the i th degradation path is led by the j th one, and $L_t^{ij} = 0$;

(3) If $E_t^{ij}(\rho \mid l < 0) = E_t^{ij}(\rho \mid l \geq 0)$, the i th degradation path leads or is led by the j th one with probability ½, i.e. $\Pr(L_t^{ij} = 1) = \Pr(L_t^{ij} = 0) = 1/2$.

Since the calculation of E_t^{ij} and E_t^{ji} is not fully symmetric considering the case of $l = 0$, it is possible that $L_t^{ij} = L_t^{ji}$. Therefore, the authors propose two criteria to revise L_t as follows.

(a) If $L_t^{ij} = L_t^{ji} = 1$, then L_t^{ij} is set as 0 if $E_t^{ij} < E_t^{ji}$ and L_t^{ji} is set as 0 if $E_t^{ij} \geq E_t^{ji}$;

(b) If $L_t^{ij} = L_t^{ji} = 0$, then L_t^{ij} is set as 1 if $E_t^{ij} < E_t^{ji}$ and L_t^{ji} is set as 1 if $E_t^{ij} \geq E_t^{ji}$.

The reason for criterion (a) is that $E_t^{ij} \geq E_t^{ji}$ means the evidence of i leading j is more powerful than that of j leading i. The reason for criterion (b) is that $E_t^{ij} \geq E_t^{ji}$ means the evidence of i being led by j is more powerful than that of j being led by i. The matrix of aggregated lagged correlation matrix E_t is revised by setting $E_t^{ij} = \max\{E_t^{ij}, E_t^{ji}\}$.

Step 4. According to Wu et al. (2010) The leadership score weights have similar relationship as those of PageRank proposed by Brin and Page (1998) and is obtained as:

$$Score_t^j = (1-d) + d \cdot \sum_{k \neq j} \frac{L_t^{jk}}{\varepsilon + \sum_{i \neq k} E_t^{ki}(1 - L_t^{ki})} Score_t^k \tag{7}$$

where $Score_t^i$ is the leadership score of degradation path i at time t, d is a damping factor which can be set between 0 and 1, and is set as 0.85 as suggested (Brin and Page, 1998), ε is a small number, say 1, to avoid zero valued denominator. Eq. (7) forms a system of linear equations, which can be solved immediately. The leadership scores are then normalized to obtain the weights for any dimensions of degradation, that is $\eta_{i,t} = Score_t^i \Big/ \sum_{j=1}^{k} Score_t^j$. The unified degradation path is then obtained as

$$D(t) = \sum_{i=1}^{k} \eta_{i,t} W_i(t) \tag{8}$$

However, due to the difference in scales of different degradation paths, the unified degradation path should be modified considering the scales. The modification is carried out by Eq. (9)

$$D(t) = \left(\sum_{j=1}^{k} \varphi_i(t) \Big/ k \right) \cdot \sum_{i=1}^{k} \left(\eta_{i,t} D_i(t) / \varphi_i(t) \right) \tag{9}$$

where $\varphi_i(t)$ is the slope for the i th degradation path at time t . Slope is used since after degradation value $D_i(t)$ being divided by slope $\varphi_i(t)$, the scale related information is eliminated from $D_i(t)$.

4. Bootstrapping based approach for reliability estimation

Due to the difficulty in deriving explicit form of reliability function for multi-dimensional degradation process, Moving Blocks Bootstrap (Efron and Tibshirani, 1993) is used to obtain the distribution of the first passage time and reliability estimate.

4.1 Bootstrapping based reliability estimation

Suppose k degradation signals are collected from sensors. The first passage time of the degradation process is defined as the time when any one of the k degradation paths reaches its critical threshold h_i . Suppose at time t , $n = t / \Delta t$ data are collected from each sensor. Thus an $n \times k$ degradation matrix is obtained. Let $b = \lceil 10\% n \rceil$ be the length of a moving block, the j th block contains the j to $j+b-1$ rows of degradation matrix. Multiple blocks are resampled from the degradation matrix and added to the end of the matrix sequentially so that first passage time is obtained for the overall degradation process. Note that the data in each block should be modified before being added to the end of the degradation matrix. First, the initial values in each resampled block should be eliminated by subtracting the row just prior to the block from each row of the block. Second, the row at the end of the present matrix should be added to each row of the block to follow. The resample process is carried out for V times to obtain V realizations of first passage time, the distribution of first passage time and the reliability estimate.

4.2 Explicit form of reliability function for Brownian motion

When a degradation path is modelled as a Brownian motion with drift, then the reliability function can be specified explicitly since the distribution of the first passage time of Brownian motion with drift coefficient λ and diffusion coefficient θ is Inverse Gaussian, and the reliability can be obtained by Eq. (10)

$$R(t) = \Phi\left(\frac{h - w_0 - \lambda t}{\theta \sqrt{t}} \right) - \exp\left(\frac{2\lambda}{\theta^2}(h - w_0) \right) \Phi\left(-\frac{h - w_0 + \lambda t}{\theta \sqrt{t}} \right) \tag{10}$$

where $\Phi(\cdot)$ is the CDF of normal distribution, h is the critical threshold and w_0 is the initial value of degradation.

For k - dimensional correlated Brownian motions, each dimension can be described as a Brownian motion with drift as specified in Eq. (2). Thus the reliability function for each individual degradation path can be calculated by Eq. (10). The corresponding parameters can be estimated using MLE and the data collected by time t . The estimates of parameters λ_i , θ_i , and $w_{i,0}$ can be obtained for each dimension respectively. For a single dimension of the multi-dimensional correlated Brownian motion process, the MLEs for the three parameters with n data are $\hat{\lambda}_i = \sum_{j=1}^{n} \Delta W_{ij} \Big/ (n\Delta t)$, $\hat{\theta}_i^2 = \sum_{j=1}^{n} (\Delta W_{ij} - \hat{\lambda}_i \Delta t)^2 \Big/ (n\Delta t)$, and $\hat{w}_{0,i} = W_{i1}$.

4.3 Explicit form of reliability function for physics-based degradation model

If a degradation path is modelled by power law degradation model combining randomness as specified by Eq. (4), the reliability can be estimated by Eq. (11) (Molini et al., 2011)

$$R(t) = \Phi\left(\frac{m(h - Y_0) - m\beta_0 t^m}{\delta m \sqrt{\beta_0 t^m}} \right) - \exp\left(\frac{2(h - Y_0)}{\delta^2} \right) \Phi\left(\frac{-m(h - Y_0) - m\beta_0 t^m}{\delta m \sqrt{\beta_0 t^m}} \right) \tag{11}$$

where the degradation path parameters m, β_0, δ, and Y_0 have the same meaning as in Eq. (4); and h is the critical threshold. The MLE parameters estimates with n data are obtained by maximizing the likelihood function in Eq. (12) and $\hat{Y}_0 = Y(0)$.

$$L(\Delta Y(t), \Delta t; \beta_0, m, \delta) = \prod_{t=\Delta t}^{n\Delta t} \frac{1}{\sqrt{2\pi}\delta\sqrt{(m\beta_0 t^{m-1})\Delta t}} \exp\left(-\frac{\left(\Delta Y(t) - (m\beta_0 t^{m-1})\Delta t\right)^2}{2\delta^2 (m\beta_0 t^{m-1})\Delta t}\right) \quad (12)$$

5. Numerical Simulation

5.1 Unified degradation path for multi-dimensional correlated Brownian motion

A numerical simulation of 4-dimensional Brownian motion process is conducted with parameters shown in Table 1 and the correlation matrix P.

Table 1: Parameter settings for 4-dimensional Brownian motion process

	Dimension 1	Dimension 2	Dimension 3	Dimension 4
μ_i	15	10	25	5
σ_i	6	3	5	2

$$P = \begin{bmatrix} 1 & 0.6 & 0.4 & 0.8 \\ 0.6 & 1 & 0.25 & 0.6 \\ 0.4 & 0.25 & 1 & 0.7 \\ 0.8 & 0.6 & 0.7 & 1 \end{bmatrix}$$

Additionally, the initial values for the four dimensions of degradation process, $w_{i,0}$, are all set as 10.

In the case of k - dimensional correlated Brownian motion, the slope of each degradation path is just the drift coefficient $\hat{\lambda}_{j,t}$ estimated at time t, the unified degradation path is modified as

$$D(t) = \left(\sum_{j=1}^{k} \hat{\lambda}_{j,t} \Big/ k\right) \cdot \sum_{i=1}^{k} (\eta_{i,t} W_i(t) / \hat{\lambda}_{i,t}) \quad (13)$$

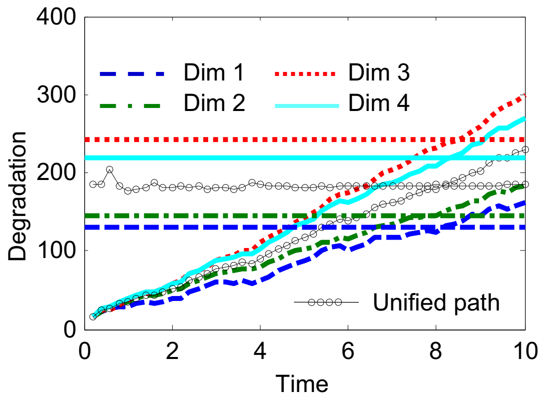

Figure 1. Unified degradation path and threshold

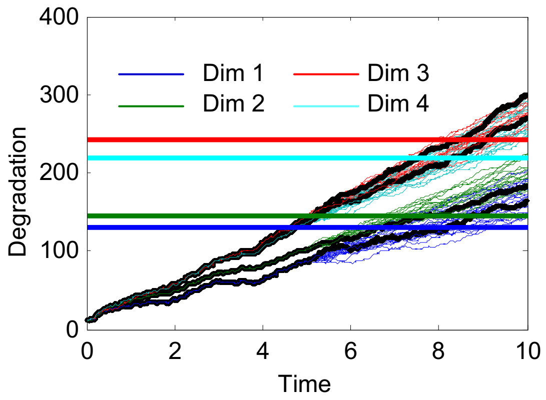

Figure 2. Moving block bootstrapping at time 5

For the k - dimensional correlated Brownian motion specified by Table 1, the unified degradation path is shown in line with circle markers in Figure 1. As can be seen from the figure, the weight-averaged threshold for the unified degradation path fluctuates around an average threshold line through time and converges due to more available data. The first passage time of the unified degradation path is approximately in the middle of the first passage times of the four single degradation paths. This shows that the unified degradation path can represent the underlying degradation process quite well.

5.2 Moving block bootstrapping method based reliability estimation

Bootstrapping based reliability information estimation is carried out at times 3, 5, and 7 respectively, with $g = 10$ and $q = \lceil 0.6(g/2) \rceil$. Figure 2 shows the bootstrapping degradation paths at time 5. The estimate for MTTF (Mean Time to Failure) becomes more accurate when more data are available. The true first passage time is 7.75, and MTTF is estimated as 6.60, 7.40 and 7.69 at times 3, 5, and 7 respectively.

The bootstrapping based reliability values at time 3 and 7 are shown in bold lines in Figure 3. From the figure it can be observed that the distribution of the first passage time, i.e. the failure time, converges to the true first passage time. The reliability values based on single degradation paths are calculated using Eq. (10) at different time points, as shown in Figure 3 in dashed lines. By comparison, it can be seen that any reliability function obtained from single degradation paths is not as accurate as the reliability obtained from k - dimensional correlated degradation paths, especially at the early stage of degradation.

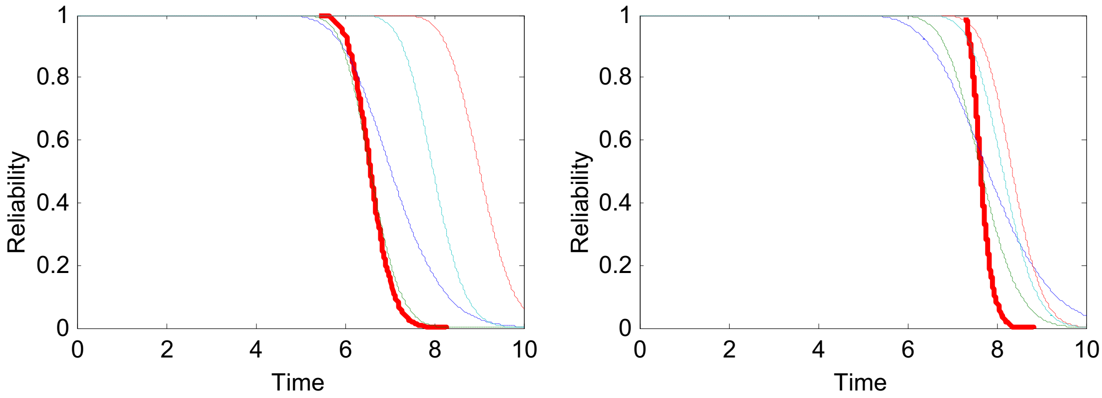

Figure 3. Reliability values at time points 3 (left figure) and 7 (right figure)

6. Conclusions

Multi-sensor degradation data analysis is studied in this paper. Multi-dimensional correlated Brownian motion and power law model (a physics-based degradation model) have been used to model multiple degradation paths. A unified degradation path is obtained by weighing multiple degradation paths, where weights are calculated by using lead-lag correlation information among different paths. The numerical simulation result shows that the unified degradation path can represent the underlying degradation quite well and hence may provide more useful information for maintenance. For multi-dimensional correlated Brownian motion based degradation data, reliability and MTTF are estimated utilizing moving block bootstrapping method. It is shown that the estimate converges with time to the true values and the reliability estimate is more accurate than using data for any of the single degradation paths.

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