

# Estimate of the State Variables of the Dynamic Behavior of the Clinker Rotary Kiln Based on a Phenomenological Approach

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Rotary kilns are multipurpose reactors, in which take place chemical and physical reactions. These reactors are used in many processes involving conversion a solid charge by heat treatment. These systems have been investigated extensively to improve the mathematical description of dynamic behaviour. Most of the research work on this type of complex system is based on models with constant or inaccessible parameters. Therefore, this present study, at the basis of a phenomenological model, offers an approach to estimate the proposed state variables of a clinker rotary kiln. This estimation is done from a steady state condition of the kiln when the system inputs were stable. Furthermore, the identification procedure developed is devoted to calculate numerically all parameters of the model, and find an analytical representation of them. Yet due to the complexity of the phenomena taking place therein, developed a dynamic model is easy of access.

## 1. Introduction

With the development of a dynamic model of Clinker Rotary Kiln (CRK), many papers have been written describing particular characteristics of the mass and energy relationship. There are many different methods employed to make a comprehensive program of energy distribution inside the CRK. First of them is represented by one dimension model based on the conservation energy (see Spang, 1972). Second, the existing model is coupled with the CFD code (see Mastorakos et al., 1999) to describe flow field of gas and solid. Furthermore, Tarasiewicz and Shahriari (2008) have shown the presence of physical and chemical phenomena strongly coupled in CRK, and then the modeling of that process requires many compromises. In addition, it is well known that the complex mathematical representation, which privilege realism of physic-chemical phenomena (see Fig. 1), cause a major problem in the identification of model parameters. Therefore, in this study it is proposed to define the state variables and regroup these unknown or inaccessible parameters in a set of parametrical functions. This reduces the quantity of unknown parameters, and a proposed model of state variables is more interesting from engineering point of view. In other words, this study is based on a phenomenological model of the process wherein each set of parameters is an operating function (OPF). So, to find the model response, the steady state operation of the kiln is taken as an initial condition. Additionally, the identification of operating functions for a dynamic behaviour was made possible by phenomenological and dimensional analyzes. Therefore, after the presentation of the model, a procedure for identifying the model parameters is described firstly. And secondly, the estimation procedure of the state variables is calculated to complete the modeling process. Finally, exposing influence of OPFs on the state variables completes the study.

## 2. The phenomenological approach

Operating functions are introduced to allow describe the physical and chemical phenomena that characterize the complexity of the system while avoiding use of complex mathematical models.

The challenge is to have an operating function that is directly identifiable from the available observations on the process.

### 2.1 The structure of the phenomenological model

The model structure is based on three state variables (Tarasiewicz and Shahriari, 2008), which are: bed material temperature  $T_c(x, t)$ , gas temperature  $T_g(x, t)$ , and the mass distribution along the kiln  $M_c(x, t)$ .

These state variables can properly describe the main phenomena taking place in the kiln (Tarasiewicz et al., 1994). The evolution of the state variables in a spatio-temporal representation of state is then expressed using a system of partial differential equations (1), (2) and (3). These equations are derived from the conservation of mass and energy as well as the physical and chemical phenomena.

The state variables, which linking the inputs and outputs, determine the dynamic evolution of the CRK as follow:

$$\frac{\partial T_g(x, t)}{\partial t} = -f_{1,1}(x, t) T_c(x, t) + f_{1,2}(x, t) \frac{\partial T_g(x, t)}{\partial x} + f_{1,3}(x, t) M_r(x, t) - f_{1,5}(x, t) (T_g(x, t) - T_c(x, t)) - f_{1,6}(x, t) (T_g(x, t) - T_b(x, t)) + f_{1,4}(x, t) \frac{\partial M_c(x, t)}{\partial x} \quad (1)$$

$$\frac{\partial T_c(x, t)}{\partial t} = f_{2,1}(x, t) T_g(x, t) + f_{2,2}(x, t) \frac{\partial T_c(x, t)}{\partial x} + f_{2,4}(x, t) (T_g(x, t) - T_c(x, t)) + f_{2,5}(x, t) (T_c(x, t) - T_b(x, t)) - f_{2,3}(x, t) \frac{\partial M_c(x, t)}{\partial x} \quad (2)$$

$$\frac{\partial M_c(x, t)}{\partial t} = -f_{3,1}(x, t) \frac{\partial M_c(x, t)}{\partial x} - f_{3,2}(x, t) T_c(x, t) + f_{3,3}(x, t) T_g(x, t) \quad (3)$$

To describe the temperature of the inner walls and the outer surface of the furnace, two additional equations are defined as follows:

$$T_A(x, t) = f_{0,3}(x, t) T_b(x, t) + f_{0,4}(x, t) T_o(x, t) \quad (4)$$

$$T_b(x, t) = f_{0,2}(x, t) T_g(x, t) + f_{0,1}(x, t) T_c(x, t) + f_{0,0}(x, t) T_A(x, t) \quad (5)$$

### 2.2 Boundary and initial conditions

To ensure that the model is well defined, it is necessary to recreate the boundary and initial conditions. This leads to the use of the sign (+) and sign (-) to designate the right and left sides of the kiln respectively.

$$\left( \dot{M}_{ap} C_{ppp} + \dot{M}_F C_{pF} \right) \frac{\partial T_g(x, t)}{\partial x} \Big|_{x=0^-} = f_{4,1}(0^+, t) (T_c(x, t) - T_g(x, t)) \Big|_{x=0^+} + f_{4,2}(0^+, t) (T_b(x, t) - T_g(x, t)) \Big|_{x=0^+} \quad (6)$$

$$\left( \dot{M}_c C_{pc} - \dot{M}_{wp} C_{ppw} - \dot{M}_{as} C_{pas} \right) \frac{\partial T_c(x, t)}{\partial x} \Big|_{x=0^-} = f_{4,3}(0^+, t) (T_c(x, t) - T_g(x, t)) \Big|_{x=0^+} + f_{4,4}(0^+, t) (T_c(x, t) - T_b(x, t)) \Big|_{x=0^+} \quad (7)$$

$$\left( \dot{M}_c C_{pc} + \dot{M}_w C_{pw} \right) \frac{\partial T_c(x, t)}{\partial x} \Big|_{x=L_T^-} = f_{5,1}(L_T^-, t) (T_g(x, t) - T_c(x, t)) \Big|_{x=L_T^+} + f_{5,2}(L_T^-, t) (T_b(x, t) - T_c(x, t)) \Big|_{x=L_T^+} \quad (8)$$

$$\Delta H_w \frac{\partial M_c(x, t)}{\partial x} \Big|_{x=L_T^+} = f_{6,1}(0^+, t) M_c(x, t) \Big|_{x=L_T^-} \quad (9)$$

The initial conditions represent the desired functionality of the CRK defined by an operator, and it is considered that the system inputs are unchanging.

### 3. Identification procedure of OPFs

It is based on phenomenological and dimensional analysis thus, at the basis of the knowledge and analysis of physic-chemical phenomena occurring in the CRK, the OPFs can be defined:

- $f_{1,1}(x, t)$  and  $f_{3,2}(x, t)$  represents the diffusion of material into the gas, so  $f_{3,2}(x, t) = \lambda_{00}(x, t)f_{1,1}(x, t)$ .
- $f_{2,1}(x, t)$  and  $f_{3,3}(x, t)$  are equivalent to the diffusion of the gas into the material, then  $f_{3,3}(x, t) = \lambda_{01}(x, t)f_{2,1}(x, t)$ .
- In view of the physic-chemical reactions of the material, which are related to the amount of heat received by them, it is reasonable to link the exchange of material  $f_{3,1}(x, t)$  with exchange of the heat  $f_{1,4}(x, t)$ ,  $f_{3,1}(x, t) = \lambda_{02}(x, t)f_{1,4}(x, t)$ .
- $f_{1,2}(x, t)$  and  $f_{2,2}(x, t)$  represents the displacements of gas, where the viscosity of the material is temperature dependence, then  $f_{2,2}(x, t) = \lambda_{03}(x, t)f_{1,2}(x, t)$ .
- By assuming that  $f_{2,4}(x, t)$  and  $f_{1,5}(x, t)$  represents the heat exchange between gas and material, then  $f_{2,4}(x, t) = f_{1,5}(x, t)$ , and can be estimated by the cited literature (see Hottel and Sarofm 1967, Tscheng 1978, and Perron 1989).
- Often in practice the combustion of the fuel gives the large quantity of the sulphur. Then  $f_{1,3}(x, t)$ , which represents the heat transfer in the gas caused by the combustion of the fuel, can be linked to the  $f_{1,4}(x, t)$ ,  $f_{1,5}(x, t)$  and  $f_{1,1}(x, t)$ , and can be written as:  $f_{1,3}(x, t) = \lambda_{04}(x, t)f_{1,4}(x, t) + \lambda_{05}(x, t)f_{1,5}(x, t) + \lambda_{06}(x, t)f_{1,1}(x, t)$ .
- If the heat exchange is related to the gas-material and material-refractory bricks then  $f_{2,1}(x, t) = \lambda_{09}(x, t)f_{1,1}(x, t)$ ,  $f_{2,5}(x, t) = \lambda_{07}(x, t)f_{2,4}(x, t)$  and  $f_{1,6}(x, t) = \lambda_{08}(x, t)f_{1,5}(x, t)$ .
- Because the heat transferred from material to gas is negligible compared to other heat transfer then  $f_{2,3}(x, t) \cong 0$ .

And the dimensional analysis, that defines the interactions between OPFs, gives the following relations:

$$\lambda_{00}(x, t) = \lambda_{01}(x, t) = \lambda_{02}(x, t) = \gamma_{ij} \frac{M_C(x, t)}{T_C(x, t)} ; \lambda_{04}(x, t) = \lambda_{10}(x, t) = \gamma_{ij} \frac{1}{\Delta x} ; \lambda_{05}(x, t) = \lambda_{06}(x, t) = \gamma_{ij} \frac{T_C(x, t)}{M_C(x, t)}$$

and  $0 < \lambda_{03}(x, t), \lambda_{07}(x, t), \lambda_{08}(x, t), \lambda_{09}(x, t), \gamma_{ij} \leq 1$

#### 4. Estimate of the state variables

The estimation of the state variables has been used when the system inputs are maintained stable. The calculations are performed over a short calculating time in order to avoid the numerical instability. Moreover, the finite difference and Runge Kutta 4 methods were used in the simulation respectively, so the finite difference formulas were used:  $\frac{\partial Y_X(x, t)}{\partial t} = \dot{Y}_{X,i}(t)$  and  $\frac{\partial Y_X(x, t)}{\partial x} = \frac{Y_{X,i}(t) - Y_{X,i-1}(t)}{\delta x}$

Applying these expressions to equations (1), (2) and (3) we obtain:

$$\begin{Bmatrix} \dot{T}_{G,i}(t) \\ \dot{T}_{C,i}(t) \\ \dot{M}_{C,i}(t) \end{Bmatrix} = A_i(t) \begin{Bmatrix} T_{G,i}(t) \\ T_{C,i}(t) \\ M_{C,i}(t) \end{Bmatrix} + A_{i-1}(t) \begin{Bmatrix} T_{G,i-1}(t) \\ T_{C,i-1}(t) \\ M_{C,i-1}(t) \end{Bmatrix} + C_i(t); A_{i-1}(t) = \begin{bmatrix} -\frac{f_{12,i-1}(t)}{\delta x} & 0 & -\frac{f_{14,i-1}(t)}{\delta x} \\ 0 & -\frac{f_{22,i-1}(t)}{\delta x} & 0 \\ 0 & 0 & \frac{f_{31,i-1}(t)}{\delta x} \end{bmatrix}$$

$$A_i(t) = \begin{bmatrix} \frac{f_{12,i}(t)}{\delta x} - f_{15,i}(t) - f_{16,i}(t) & -f_{11,i}(t) + f_{15,i}(t) & \frac{f_{14,i}(t)}{\delta x} \\ f_{21,i}(t) + f_{24,i}(t) & \frac{f_{22,i}(t)}{\delta x} - f_{24,i}(t) + f_{25,i}(t) & 0 \\ f_{33,i}(t) & -f_{32,i}(t) & -\frac{f_{31,i}(t)}{\delta x} \end{bmatrix};$$

$$C_i(t) = \begin{Bmatrix} f_{13,i}(t)M_{F,i}(t) + f_{16,i}(t)T_{B,i}(t) \\ -f_{25,i}(t)T_{B,i}(t) \\ 0 \end{Bmatrix}$$

By varying  $i$  from 0 till  $n$ , the following matrix of the first order of the differential equation is obtained:

$$\dot{Y}(t) = A(t)Y(t) + C(t) \tag{10}$$

$Y(t) = (y_1; y_2; \dots; y_n)^T$ ,  $y_i(t) = (T_{G,i}(t); T_{C,i}(t); M_{C,i}(t))^T$ ,  $C(t) = (C_{01}; C_2; \dots; C_{n-1}; C_n)^T$ , with

$$A(t) = \begin{bmatrix} A_1 & 0 & 0 & \dots & \dots & \dots & \dots & 0 & 0 \\ A_1 & A_2 & 0 & \dots & \dots & \dots & 0 & 0 & 0 \\ 0 & A_2 & A_3 & 0 & \dots & \dots & 0 & 0 & \dots \\ \vdots & 0 & \dots \\ \vdots & \vdots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & \vdots & \dots & 0 & A_{n-3} & A_{n-2} & 0 & 0 & 0 \\ 0 & \vdots & \dots & \dots & 0 & A_{n-2} & A_{n-1} & 0 & 0 \\ 0 & 0 & \dots & \dots & 0 & 0 & A_{n-1} & A_n & 0 \end{bmatrix}$$

and  $C_{01} = C_1 + A_0 \cdot y_0$

Finally, using Eq. 10, the state variables have been estimated as follows (see Figs. 2, 3, and 4).

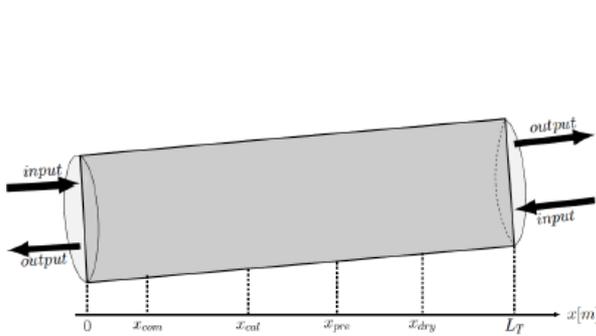


Figure 1: Zones distribution in the CRK

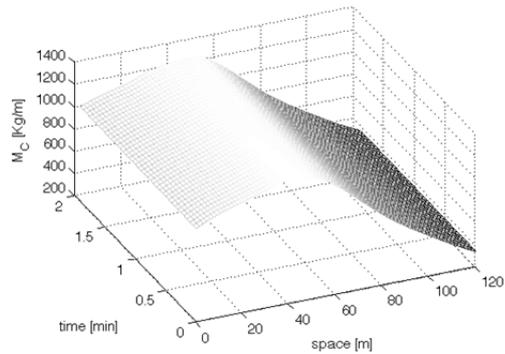


Figure 2: Evolution of estimated  $M_C(x, t)$

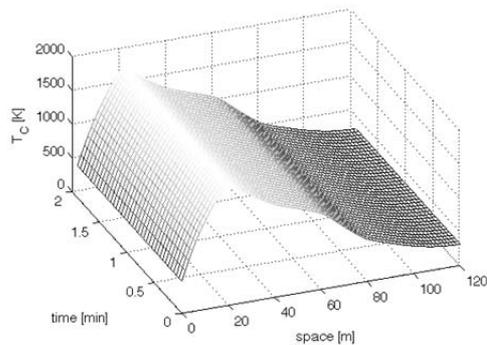


Figure 3: Evolution of estimated  $T_C(x, t)$

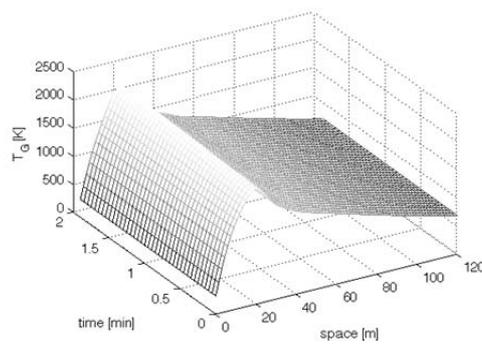


Figure 4: Evolution of estimated  $T_G(x, t)$

Where:  $[0^+; x_{com}^-]$ : recovery zone;  $[x_{com}^+; x_{cal}^-]$ : combustion zone;  $[x_{cal}^+; x_{pre}^-]$ : calcination zone;  $[x_{pre}^+; x_{dry}^-]$ : preheating zone; and  $[x_{dry}^+; L_T^-]$ : drying zone.

#### 4.1 Influence of OPFs on the state variables

To illustrate the influence of the OPFs on the state variables, consider the situation that only one operating function is involved, it is interesting to compare the shapes of the state variables which are expressed by Figures 2, 3, and 4 and Figures 5, 6, and 7. This function  $f_{1,1}(x, t)$  is required to be performed not only during normal operating condition of the CRK, but also during some perturbation. A review of the test of the numerical simulations (Tarasiewicz and N'zi, 2012) it is possible to recognize that the proposed OPFs reflect very well real-situation inside the kiln.

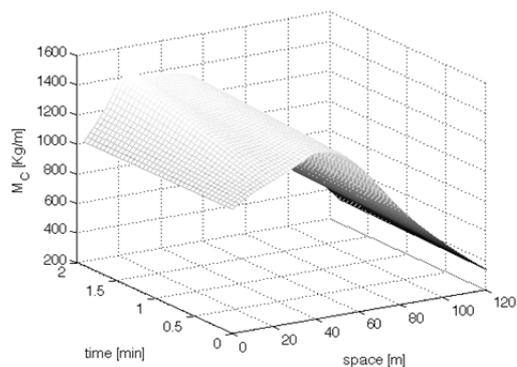


Figure 5: Influence of  $M_c(x, t)$  by  $f_{1,1}(x, t)$

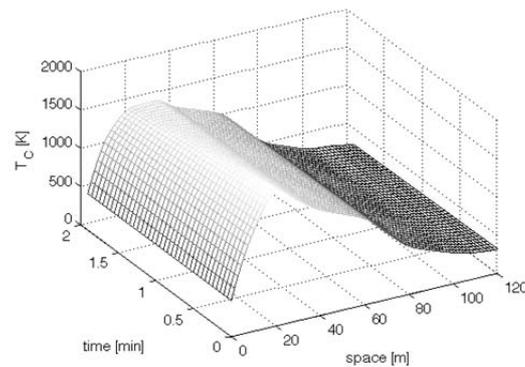


Figure 6: Influence of  $T_c(x, t)$  by  $f_{1,1}(x, t)$

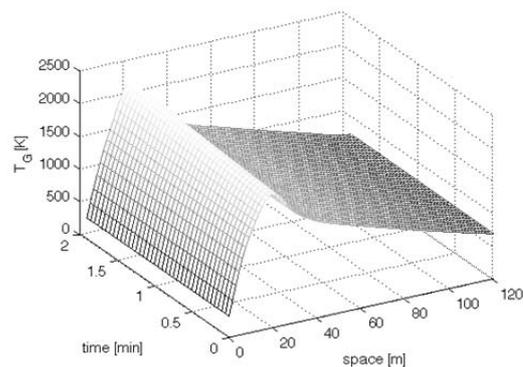


Figure 7: Influence of  $T_g(x, t)$  by  $f_{1,1}(x, t)$

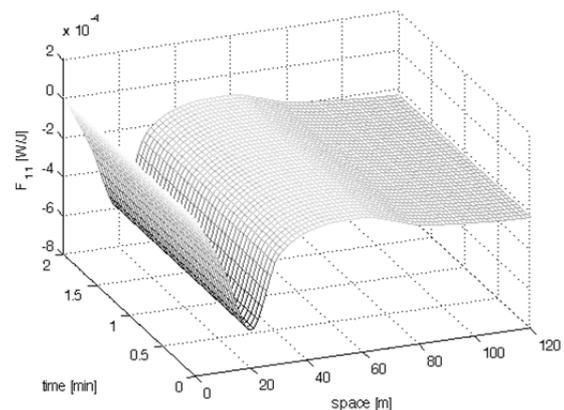


Figure 8: Evolution of  $f_{1,1}(x, t)$

Where: the shape of  $f_{1,1}(x, t)$  is represented in Figure 8.

It has been demonstrated that an understanding of each operating function (Tarasiewicz and N'zi, 2012) and its influence on the state variables is crucial to the proper formulation of standard testing instrumentations. However, the intention of minimal number of measured system is extremely difficult to establish to the real-time investigation. Because, developing a dynamic model practical enough for computer application on control is no simple undertaking.

## 5. Conclusion

This study has shown that the proposed model is useful in the determination of all OPFs and its effect on the state variables profiles. For a given situation and a desired initial condition profiles, a good value of boundary conditions can be found. In a situation where some or all OPFs are predetermined by the steady-state condition of the CRK, the model helps to find the best values of all OPFs and boundary condition in function of time and space. In further works now being pursued, the authors apply the same model to propose a new version of the real-time optimal control.

## Nomenclature

$\dot{M}$ : Mass flow rate, [kg/s]  
 $M$ : Distributed mass, [kg/m]  
 $C_p$ : Specific heat, [J/kg · K]  
 $\Delta H$ : Heat of reaction latent heat, [J/kg]  
 $T$ : Temperature, [K]  
 $f_{i,j}, F_{i,j}$ : operating function  
 $L_T$ : kiln length, [m]

## Subscript

$w$ : Water  
 $as$ : Secondary air  
 $ap$ : Primary air  
 $wp$ : Water vapor  
 $f, F$ : Fuel  
 $A, B, O, G, C$ : Outer surface of the kiln, refractory bricks, ambient air, gas, and material

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