

## The Heat and Momentum Transfers Relation in Channels of Plate Heat Exchangers

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The link between heat transfer intensity and hydraulic resistance of PHE channels is determined with the use of modified Reynolds analogy of heat and momentum transfer. The formula to estimate the share in total hydraulic resistance of pressure loss due to friction is proposed. The resulting model enables to calculate film heat transfer coefficients in PHE channels on a data of hydraulic resistance of the main heat transfer field. The calculations are compared with the available in literature experimental results on heat transfer in channels with cross corrugated walls. The good agreement confirms the assumptions made on deriving equation and existence in PHE channels of the analogy between heat and momentum transfer in proposed form.

### 1. Introduction

Sustainability in the process industry requires efficient heat recuperation, as it is shown by Klemeš et al. (2011). Plate Heat Exchangers (PHEs) are now widely used in different industrial applications. They save a lot of space and construction material, increase reliability and operability compare to shell and tubes heat exchangers. The principles of their construction and design methods are sufficiently well described in literature (see e.g. Wang et al. (2007)). The thermal and hydraulic performance of PHE mainly determined by intensified heat transfer processes in their channels of complex geometry, which are formed by corrugated plates pressed from thin sheet metal. The link between heat and momentum transfer in channels with enhanced heat transfer is very important for selection of the optimal geometry and correct design of heat exchangers, see Kukulka and Fuller (2010).

Dović et al. (2009) used Leveque equation to generalize heat transfer data for PHEs, published by different authors. The accuracy of prediction is rather good for some cases, but the error by their estimation sometimes reach up to 40 - 52%. This can be partly explained by attempt to generalize together the data for models of corrugated fields of PHE channels and data for commercial plates. As it can be judged from the data of Tovazhnyansky et al. (1980), the share of pressure drop in entrance and exit zones of some PHE channels, formed by commercial plates of chevron type, can reach 50% and more, especially at lower angles of corrugations inclination to plate vertical axis. The other reason can be seen in accuracy of the employed relation between heat transfer and

hydraulic resistance. In our study we performed data generalization on a base of the data on hydraulic resistance for PHE channel main corrugated field.

## 2. Modification of Reynolds analogy for PHE channels

Let's assume that relationship between heat transfer and frictional shear stress on the channel wall is in PHE channels the same as in tubes. In a tubes Reynolds analogy holds true and, as was shown by Tovazhnyansky and Kapustenko (1984b), the following equation can be derived for PHE channel:

$$Nu = 0.065 \cdot Re^{6/7} \cdot (\psi \cdot \zeta_s)^{3/7} \cdot Pr^{0.4} \cdot \left( \frac{\mu}{\mu_w} \right)^{0.14} \quad (1)$$

Here  $\mu$  and  $\mu_w$  dynamic viscosity at stream and at wall temperatures;  $Nu = h \cdot d_e / \lambda$  - Nusselt number;  $\lambda$  - heat conductivity of the stream, W/(m·K);  $d_e$  - equivalent diameter of channel, m;  $h$  - film heat transfer coefficient, W/(m<sup>2</sup>·K); Pr - Prandtl number;  $\zeta_s$  - friction factor accounting for total pressure losses in channel;  $\psi$  - the share of pressure loss due to friction on the wall in total loss of pressure;  $Re = w \cdot d_e / \nu$  - Reynolds number;  $w$  - stream velocity in channel, m/s;  $\nu$  - cinematic viscosity, m<sup>2</sup>/s. For calculation of friction factor  $\zeta$  for the main corrugated field of PHE channels we use the correlation proposed by Arsenyeva (2010). It enables to predict  $\zeta$  for a wide range of chevron plates corrugation parameters. This correlation is as follows:

$$\zeta = 8 \cdot \left[ \left( \frac{12 + p2}{Re} \right)^{12} + \frac{1}{(A + B)^2} \right]^{\frac{1}{12}}; \quad A = \left[ p4 \cdot \ln \left( \frac{p5}{\left( \frac{7 \cdot p3}{Re} \right)^{0.9} + 0.27 \cdot 10^{-5}} \right) \right]^{16}; \quad B = \left( \frac{37530 \cdot p1}{Re} \right)^{16} \quad (2)$$

where p1, p2, p3, p4, p5 – parameters defined by channel corrugation form.

$$p1 = \exp(-0.15705 \cdot \beta); \quad p2 = \frac{\pi \cdot \beta \cdot \gamma^2}{3}; \quad p3 = \exp\left(-\pi \cdot \frac{\beta}{180} \cdot \frac{1}{\gamma^2}\right), \quad (3)$$

$$p4 = \left( 0.061 + \left( 0.69 + \operatorname{tg}\left(\beta \cdot \frac{\pi}{180}\right) \right)^{-2.63} \right) \cdot (1 + (1 - \gamma) \cdot 0.9 \cdot \beta^{0.01}), \quad p5 = 1 + \frac{\beta}{10}$$

Here  $\gamma = 2b/S$  – the corrugation profile aspect;  $\beta$  – the corrugation angle to the plate longitudinal axis;  $b$  – corrugation height, m;  $S$  – pitch of corrugations, m.

The friction factor in Eq.(2) assumes the equivalent diameter of the channel as  $d_e = 2b$  and characteristic length – the length of the channel  $L$ . Respectively, the Reynolds and Nusselt numbers in Eq.(1) must be determined at  $d_e = 2b$  and the friction factor from Eq.(2) must be divided on the ratio  $Fx$  of actual surface area to projected one:  $\zeta_s = \zeta / Fx$ . It is made to account for distribution of shear stresses around all effective heat transfer surface area, similar to distribution of film heat transfer coefficient. For accurate comparison with experimental data it should be taken into consideration that some researchers are taking hydraulic diameter in reduction of their experimental data. The main parameter, which is not determined in Equation (1) is the share of pressure loss due to friction on the wall in total loss of pressure  $\psi$ . Its value can be estimated on

comparison with experimental data on heat transfer in models of PHE channels. On Figure 1 are presented the experimental data of Tovazhnyansky et al. (1980) for four experimental samples of PHE channel. The parameters of the samples are given in Table 1. The Nusselt and Reynolds numbers are recalculated for  $de=2b$ .

Table 1: Geometrical parameters of experimental samples

Sample No	Pitch S, mm	Height b, mm	$\beta$ , degrees	Length L, m	Width W, mm	$d_h$ , mm	$F_x$
1	18	5	60	1.0	225	9.6	1.15
2	36	10	60	1.0	225	19.3	1.15
3	18	5	45	1.0	225	9.3	1.15
4	18	5	30	1.0	225	9.0	1.15

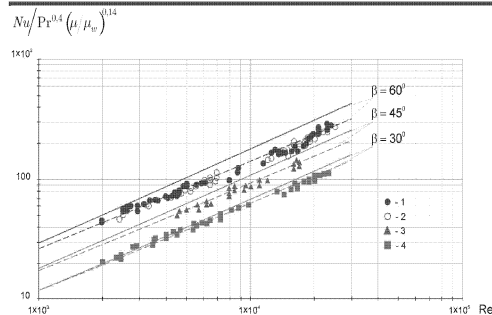


Figure 1: Heat transfer in samples of PHE channels  $\gamma = 0.556$

(lines are calculated by Eq.(1):

—  $\psi=1$ ; - - -  $\psi$  by Eq.4).

1, 2, 3, 4 – samples numbers, see Table 1

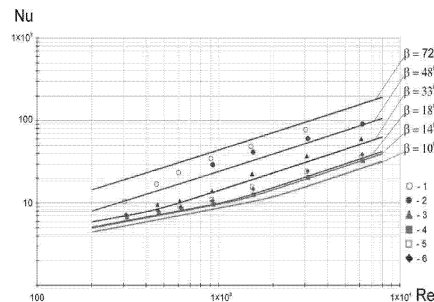


Figure 2: Comparison with data of Savostin and Tikhonov (1970):

1 -  $\beta = 72^\circ$ ,  $\gamma = 0.872$ ; 2 -  $\beta = 48^\circ$ ,  $\gamma = 0.926$ ; 3 -  $\beta = 33^\circ$ ,  $\gamma = 0.911$ ;  
4 -  $\beta = 18^\circ$ ,  $\gamma = 0.926$ ; 5 -  $\beta = 14^\circ$ ,  $\gamma = 1.460$ ; 6 -  $\beta = 10^\circ$ ,  $\gamma = 0.934$ .

Analysis of data presented on Fig.1 lead to following conclusion. The discrepancies of calculations by Equation (1) at  $\psi=1$  with experimental data are rising with the increase of corrugations angle  $\beta$  and Reynolds number. The analysis of flow patterns in PHE channels by Dović et al. (2009) have shown a stronger mixing at higher  $\beta$  and Reynolds numbers. The mixing is associated with flow disruptions, which are contributing to the rise of form drag and consequent decrease in the share  $\psi$  of pressure loss due to friction on the wall. Correlating  $\psi$  calculated from Eq.(1) at experimental values of  $Nu$ , the formula describing its dependence from  $\beta$  and Reynolds number is obtained:

$$A = 380 / [\operatorname{tg}(\beta)]^{1.75}; \text{ at } Re > A \quad \psi = \left( \frac{Re}{A} \right)^{-0.15 \cdot \sin(\beta)}; \text{ at } Re \leq A \quad \psi = 1 \quad (4)$$

The Eq (1) with  $\psi$  calculated by Eq.(4) correlates experimental data on Fig.1 with mean square error 6.2 %. The Equations (1) – (4) represent the mathematical model describing the influence of the corrugations geometrical parameters on heat transfer. Considering that the flow distribution zones of PHE plates have not more than 15-20 % of the total plate heat transfer area, they have much less influence on the total heat

transfer. A reasonable accuracy of presented mathematical model also for industrial PHEs, not only for models of the corrugated field, can be expected

### 3. Comparison of the model with experiments

The comparison with data published by Savostin and Tikhonov (1970) is presented on Fig.2. The data were obtained with air at temperatures 368 and 463 K, for models of channels with corrugated walls. We assumed  $Pr=0.69$  and  $(\mu/\mu_w)^{0.14}=1$ . The corrugations had triangular shape with radiuses (about 0.6 mm) at the edges. The height of corrugations for the cases on Fig.2 was in the range 1.12-1.22 mm. The experimental Nu and Re were multiplied on  $2b/d_h$ , as to make comparison at the same definition of equivalent diameter. The discrepancies for  $\beta=14^\circ$ ,  $18^\circ$ ,  $33^\circ$  and  $48^\circ$  not exceed 15 %. For  $\beta=10^\circ$  the error up to 25 % and it can be concluded that lower limit is  $\beta=14^\circ$ . The upper limit is certainly below  $72^\circ$ , as discrepancies at  $\beta=72^\circ$  going up to 50 % at  $Re < 800$ .

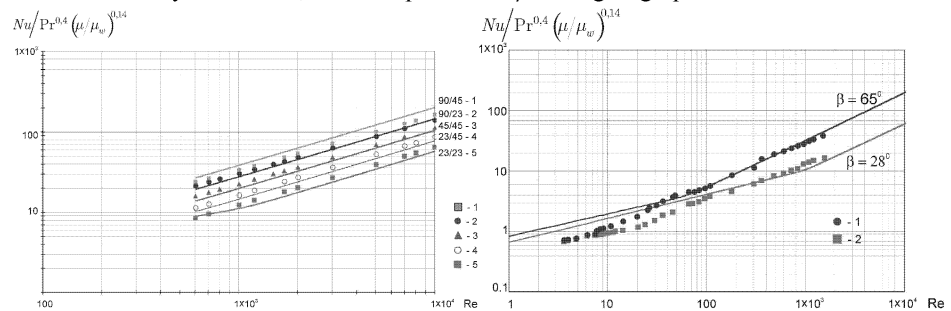


Figure 3: Comparison with data of Heavner et al. (1993):

1 -  $\beta=(90^\circ+45^\circ)/2$ ; 2 -  $\beta=(90^\circ+23^\circ)/2$ ;  
3 -  $\beta=45^\circ$ ; 4 -  $\beta=(23^\circ+45^\circ)/2$ ;  
5 -  $\beta=23^\circ$ .

Figure 4: Comparison with heat transfer data of Dović et al. (2009):

1 -  $\beta=65^\circ$ , 2 -  $\beta=28^\circ$   
( $\gamma=0.52$ ,  $Fx=1.19$ ).

Another comprehensive study of heat transfer with extended set of mixed plates of different corrugations was reported by Heavner et al. (1993). These data are presented on Figure 3 by points calculated from correlations presented in book by Wang et al. (2007). The equations were obtained for the exponent at Pr equal to 1/3. To account for differences with exponent in Eq.(1), which is equal to 0.4, the data were corrected on 0.91. It is made approximately for average Prandtl number with experiments on water equal to 4:  $4^{1/3}/4^{0.4}=0.91$ . The estimation for geometrical parameters was taken from the paper of Dović et al. (2009):  $\gamma=0.7$ ,  $Fx=1.26$ . The results of calculation by mathematical model (1) – (4) are presented by solid lines on Figure 3. The agreement with experiment is rather good for  $\beta$  from 23 to 23/90 degrees. The discrepancies not exceed 10 %. For the highest  $\beta$  at combination of plates with  $\beta=90^\circ$  and  $\beta=45^\circ$  (average  $\beta=67.5^\circ$ ) the experimental data are from 14 to 23 % lower, than predicted by model. The research for  $\beta=65^\circ$  was reported by Dović et al. (2009). They have performed tests with two models of PHE channels having corrugations at the angles  $\beta=65^\circ$  and  $\beta=28^\circ$ . The tests were made with water and water – glycerol solutions. Water data situated at Re higher than 200. Heat transfer results of these authors are presented in Figure 4. As they used hydraulic diameter  $d_h=2b/Fx$ , the values of Nu and Re were corrected on  $Fx=$

1.19 as multiplier. Same correction as for data on Figure 3 was made to account for difference in Pr exponent. The data were taken from a graph, so the accuracy is limited, but for  $Re > 100$  we estimate model prediction (solid lines on Figure 4) as fairly good. The error for  $\beta = 65^\circ$  not exceed 10 %. We can judge that the upper  $\beta$  - limit of model application as  $\beta = 65^\circ$ . The lower limit for Reynolds number can be estimated as 100.

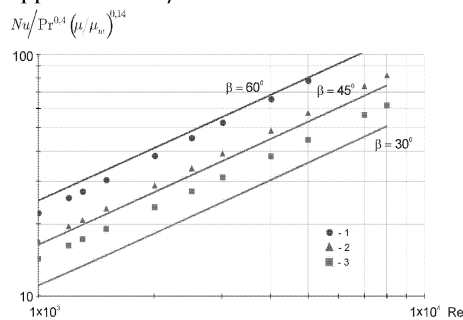


Figure 5: Comparison with data of Muley and Manglik (1999): 1 -  $\beta = 60^\circ$ , 2 -  $\beta = (60^\circ + 30^\circ)/2$ , 3 -  $\beta = 30^\circ$  ( $\gamma = 0.556$ ,  $Fx = 1.29$ ).

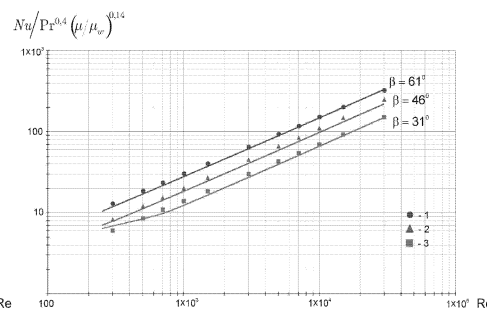


Figure 6: Comparison with data of Arsenyeva et al. (2009): 1 -  $\beta = 61^\circ$ , 2 -  $\beta = (61^\circ + 31^\circ)/2$ , 3 -  $\beta = 31^\circ$  ( $\gamma = 0.588$ ,  $Fx = 1.21$ ).

The data of heat transfer and friction factor for different arrangements of two plates with  $\beta = 60^\circ$  and  $\beta = 30^\circ$  are from Muley and Manglik (1999). Their tested for cooling of hot water ( $2 < Pr < 6$ ) and used equivalent diameter  $de = 2b$ . The exponent at Pr was taken  $1/3$ , so we have made correction like for data on Figures 3 and 4. The predictions by the model (solid lines on Fig.5) are rather good for angles  $\beta = 60^\circ$  and combination of plates with  $\beta = 60^\circ$  and  $\beta = 30^\circ$  (average  $\beta = 45^\circ$ ), the error not bigger than 10 %. But for low angle  $\beta = 30^\circ$  the model underestimates data on up to 30%. It can be explained by the influence of inlet and outlet distribution zones of the short tested plates (the distance between the ports 392 mm, the length of the main corrugated field approximately  $Lp = 280$  mm). While the area of distribution zones is about 20% of the total heat transfer area, the higher level of generated at entrance turbulence influence the heat transfer on all plate, especially at low  $\beta$ , when there is about 15 cells formed by corrugations on a length of a channel. For  $\beta = 60^\circ$  the number of such cells is 27 and for  $\beta = 45^\circ$  – 23 cells. For this plate the ratio  $Lp/de = 55$ , but we can expect that such effect will be eliminated at  $Lp/de > 100$ . The data for heat transfer of longer plates were reported by Arsenyeva et al. (2009). These data for commercial plates M10B of Alfa-Laval production are presented on Fig.6. The length  $Lp$  is 720 mm and  $Lp/de$  about 120. The error of model did not exceed 10% for all  $\beta$ , which were 1)  $\beta = 61^\circ$ , 2) mixed  $\beta = 61^\circ$  and  $\beta = 31^\circ$  (average  $46^\circ$ ), 3)  $\beta = 31^\circ$ . It confirms the accuracy of developed model for commercial plates.

#### 4. Conclusions

The presented mathematical model enables to predict the film heat transfer coefficients in PHE channels on a data of their corrugations geometrical parameters, such as

corrugation angle  $\beta$ , aspect ratio  $\gamma$  and coefficient of surface area enlargement  $F_x$ . The comparison with available in literature data have shown that the error of heat transfer prediction by proposed model is not larger than 15% in the following range of corrugations parameters:  $\beta$  from 14° to 65°;  $\gamma$  from 0.5 to 1.5;  $F_x$  from 1.14 to 1.5. This is confirmed for Reynolds numbers from 100 to 25000 for both shapes of individual corrugations: sinusoidal and triangular with rounded edges. The comparison with data for some commercial plates gave good results also. But for angles  $\beta$  lower than 30° the limitation should be made for the ratio of plate length to double spacing less than 100.

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