

## Liquid Dispersion Analysis in Fabrics - Flash MMT Method

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This work presents a new method called “flash MMT method” developed for the quantitative measurements of liquid dispersion coefficients in fabrics. Its principle is based on the combination of the well known flash method used for the measurement of thermal diffusivities of materials and the Moisture Management Tester (MMT) method used for qualitative characterization of liquid spreading in fabrics.

It was shown that when the liquid injection on the top surface of a fabric follows a Dirac function, the flash MMT 2D model developed has an analytical solution where lateral and axial dispersion coefficients are involved as unknown parameters. The latter were then deduced from experimental measurements of water content on both top and bottom surfaces of the fabric. The agreement between the model predictions and the experimental measurements of water content is quite encouraging even though the liquid injection part of the method deserves to be improved.

### 1. Introduction

In the sportswear design, among the most influencing parameters of the fabric used, lateral (on both faces) and axial dispersion coefficients of the liquid are of great importance. Their experimental measurements and theoretical predictions are therefore challenging.

Recently, a new method and instrument named Moisture Management Tester (MMT) has been developed and used to determine the liquid spreading and transfer rates of fabrics (Hu et al, 2005). Although the results obtained are quite interesting, it is important to notice that only qualitative information can be determined with the MMT. The measurements do not contain the information which is able to quantify the dispersion coefficients. To overcome this problem, we combined the flash method with the MMT in what we called “Flash MMT” method.

The flash method was mainly developed for the measurement of heat dispersion coefficients of materials. It consists in imposing a pulse of heat during a very short time (flash) on one of the faces of a cylindrical sample of material. The temperatures of the material are then measured on the same face and on the opposite face as well. The heat dispersion coefficients are then deduced from the comparison of the measurements to the heat model predictions (Parker et al, 1961; Lachi, 1991; Degiovanni et al, 1996; Demange et al, 1997).

In the present communication, the flash method is adapted to mass transport measurements in fabrics. Instead of a heat pulse in thermal measurements, a pulse of water is used. The MMT is then used for the measurements of water content on the two faces of the fabric. The mass dispersion coefficients are then deduced from the comparison of the measurements of water content to the predictions of a transient 2D model (Quiniou, 2009).

## 2. Process Model

We consider a cylindrical sample of a fabric with a thickness of  $L_z$  and a diameter of  $L_r$ . Water is injected on the upper surface of the fabric corresponding to the coordinate  $z=L_z$  over a diameter  $R_{inj}$  less than  $L_r/2$ . The model is developed assuming that the water injection is close to a Dirac function under isothermal conditions. Gravity effects are neglected since the value of GS number (Li and Zhu, 2003) estimated in our experimental conditions is around 0.1. The transient 2D diffusion equation of water in the fabric is then expressed as

$$\frac{\partial(\rho\varepsilon)}{\partial t} = \frac{\partial}{\partial r} \left( D_r \frac{1}{r} \left( \frac{\partial(r\rho\varepsilon)}{\partial r} \right) \right) + \frac{\partial}{\partial z} \left( D_z \left( \frac{\partial(\rho\varepsilon)}{\partial z} \right) \right) \quad (1)$$

where :  $0 < t \leq t_f$ ,  $0 < r < \frac{L_r}{2}$ ,  $0 < z < L_z$ .  $\varepsilon$  and  $\rho$  are the volume fraction and density of the water liquid respectively,  $D_r$  and  $D_z$  are the lateral and axial dispersion coefficients respectively .

The boundary conditions are defined assuming that there is no mass exchange with the surrounding.

As in the thermal flash method, the initial condition must be represented by a Dirac function in order to ease the solution of the model equations using the variable separation method. Thus, at the initial time ( $t=0$ ), the volume fraction of the liquid water in the fabric must be expressed by the product of an axial term and a lateral term as

$$\varepsilon(r, z, 0) = f_0(r) \cdot g_0(z) \quad f_0(r) = \begin{cases} 1 & \text{if } 0 \leq r \leq R_{inj} \\ 0 & \text{if } r > R_{inj} \end{cases} \quad g_0(z) = \begin{cases} \varepsilon_{inj} & \text{if } 0 \leq z \leq e \\ 0 & \text{if } z > e \end{cases}$$

This means that the water injected on the fabric surface during the experiment is initially located in the cylindrical zone defined by the radius  $R_{inj}$  and the thickness  $e$ . In that zone, at the initial time, the liquid volume fraction is  $\varepsilon_{inj}$  and zero in the rest of the fabric.

According to the variable separation method, the solution is expressed as

$$\varepsilon(r, z, t) = A(z, t) \cdot R(r, t) \quad (2)$$

where the axial and lateral terms are given by the solution of the following partial differential equations :

$$\frac{\partial^2 A}{\partial z^2} = \frac{1}{D_z} \frac{\partial A}{\partial t} \quad \text{and} \quad \frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} = \frac{1}{D_r} \frac{\partial R}{\partial t} \quad (3)$$

The variable separation method is used again here to solve each of the two equations. The calculus details are developed in (Quiniou, 2009). The analytical solution of the model equation with associated initial and boundary conditions is expressed as

$$\varepsilon(r, z, t) = \left[ A_0 + 2A_0 \sum_{k=1}^{+\infty} \cos\left(\frac{k\pi z}{L_z}\right) \exp\left(-D_l^z \frac{k^2 \pi^2}{L_z^2} t\right) \right] \times \left[ B_0 + \sum_{k=1}^{+\infty} B_k J_0\left(\alpha_k \frac{2r}{L_r}\right) \exp\left(-D_l^r \frac{4\alpha_k^2}{L_r^2} t\right) \right] \quad (4)$$

with  $A_0 = \frac{Me_0}{\rho\pi R_{inj}^2 L_z}$ ,  $B_0 = \frac{4R_{inj}^2}{L_r^2}$ ,  $B_k = \frac{4R_{inj}}{\alpha_k L_r} \frac{J_1(2\alpha_k R_{inj}/L_r)}{J_0^2(\alpha_k)}$ , and  $\alpha_k$  the k-th root of  $J_1$ .

It can be seen that the solution involves two infinite series which have to converge in order to compute the value of liquid volume fraction at  $r$ ,  $z$  and  $t$ . It is important to notice that the convergence rate of the series slows down when the time approaches the beginning of the experiment.

### 3. Experimental Measurements

Here the Moisture Manager Test apparatus (Hu et al, 2005) is used to determine the liquid spreading and transfer rates of a fabric. The principle of the method is based on the change of the electrical resistance of the fabric with its water content. Six concentric rings (sensors) of different sizes are then placed on both surfaces of the fabric. The distance between two consecutive rings is 5 mm except the first one which is at 1.5 mm from the centre. They allow us to measure the spreading and transfer of a 0.22 g drop of water during 2 minutes. More specifically they allow to determine the water content (WC) at different ring locations (local) and consequently on the overall (global) surface. WC is the result of integration, over a ring, of the ratio between the weight of free water and the weight of dry fabric, given as

$$WC_i^{j,th} = \frac{2\pi}{S_i^j} \int_{r_i}^{r_{i+1}} \frac{\rho\varepsilon}{\rho_f \varepsilon_f} r dr \quad (5)$$

$i$  ( $=1,2,3,4,5,6$ ) refers to a ring and  $j$  ( $=\text{top, bottom}$ ) to a surface.  $\varepsilon_f$  and  $\rho_f$  are the volume fraction and density of the fabric respectively and  $S$  is the fabric surface bounded by a ring.

### 4. Process Model Parameters Identification

The process model developed and solved analytically involves two unknown parameters, i.e. axial ( $D_z$ ) and lateral ( $D_r$ ) liquid dispersion coefficients. Their optimal values will be deduced from the comparison of the measured values of water content using the MMT apparatus and those predicted using the process model.

The identification procedure is as follows. For two (top and bottom) rings located at the same radius  $i$ , the ratio of predicted water contents is given as

$$\frac{WC_i^{bot,th}}{WC_i^{top,th}} = \frac{A(L_z, t)}{A(0, t)} \quad (6)$$

From Eq.(4), it can be shown that the only unknown parameter involved in the above ratio is the axial dispersion coefficient  $D_z$ . This parameter can be deduced from the minimisation of the following objective function:

$$F(D_z) = \frac{1}{N_{mes}} \sum_{n=1}^{N_{mes}} \left[ \frac{WC_i^{bot,exp}(t_n)}{WC_i^{top,exp}(t_n)} - \frac{WC_i^{bot,th}(t_n)}{WC_i^{top,th}(t_n)} \right]^2 \quad (7)$$

which is the sum of least squares between the measured and predicted ratios of water contents using two rings located at the same radius.

On the other hand, for two neighbouring rings located at the same (top or bottom) surface, the ratio of predicted water contents is expressed as

$$\frac{WC_{i_2}^{j,th}}{WC_{i_1}^{j,th}} = \frac{S_{i_1}^j \int_{r_{i_2}}^{r_{i_2+1}} \rho R(r, t) r dr}{S_{i_2}^j \int_{r_{i_1}}^{r_{i_1+1}} \rho R(r, t) r dr} \quad (8)$$

Here also, it can be shown (from Eq.(4)) that the only unknown parameter involved in the above ratio is the axial dispersion coefficient  $D_r$ . This parameter can be deduced from the minimisation of the following objective function:

$$F(D_r) = \frac{1}{N_{mes}} \sum_{n=1}^{N_{mes}} \left[ \frac{WC_{i_2}^{j,exp}(t_n)}{WC_{i_1}^{j,exp}(t_n)} - \frac{WC_{i_2}^{j,th}(t_n)}{WC_{i_1}^{j,th}(t_n)} \right]^2 \quad (9)$$

which is the sum of least squares between the measured and predicted ratios of water contents using two neighbouring rings located at the same surface.

## 5. Results and Discussions

It is important to mention that the computation time needed to get water content at different rings is quite large. Instead, we used a CFD software, i.e. Comsol Multiphysics®, to simulate the flash MMT apparatus. The analytical solution developed above was used to validate the numerical solution whose computation time is almost instantaneous.

The lateral and axial dispersion coefficients  $D_r$  and  $D_z$  are identified using the optimisation code *fmincon* available within the Optimisation Toolbox of Matlab® environment. This was made possible through a link between Matlab and Comsol Multiphysics. The latter was called within Matlab interface. The 95% confidence interval of each parameter was then computed in order to give an idea about the accuracy of its identification. The results are shown in Table 1 where the corresponding value of the objective function is reported.

Table 1: Identification and confidence intervals for flash MMT experiments

Flash MMT	$D_r$	$D_z$	$F$
Value	$2.4 \times 10^{-7}$	$4.7 \times 10^{-7}$	$5.5 \times 10^3$
95% Confidence interval (%)	$4 \times 10^{-2}$	4.1	

Since the water is injected on the top surface and diffuses to the bottom surface of the fabric, the axial dispersion coefficient is two times greater than the lateral one. It should be noticed that this is true for the experimental protocol used. The same result is not guaranteed for a different experimental protocol. The computed confidence intervals show that the parameters were determined with a quite high accuracy.

Top of figure 1 shows the time-varying of the overall water content obtained by adding the water content of each ring on both top and bottom fabric surfaces, whereas the two bottom graphs in figure 1 show the total water content on the top and bottom surfaces respectively. The total top and bottom water content are obtained by summing up the water content of the rings at the top and bottom fabric surface respectively.

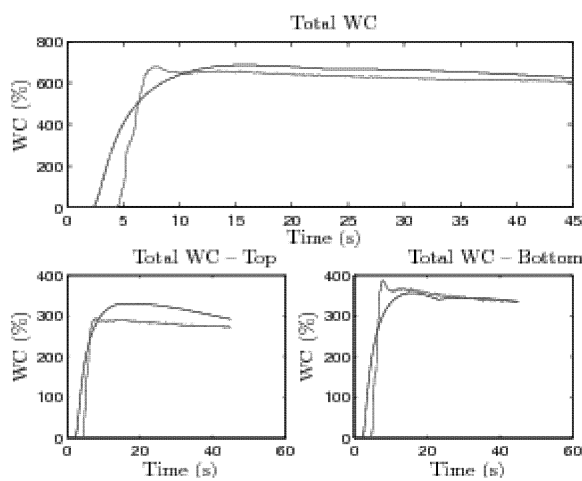


Figure 1: Comparison between experimental and theoretical results

It can be seen that after a period of 10 s corresponding to the injection period (injection duration and fabric's response) the measured values of WC are in good agreement with those predicted by the model. This result is quite expected since the injection duration has not been accounted for in the model since it is considered as instantaneous. This is obviously one of the limits of the flash MMT method implemented in this work and deserves to be improved. On the other hand better results are obtained when total top and bottom WC are considered. However, when WCs of the first two rings are considered, the agreement between the measured and predicted values is quite poor particularly for the second ring (not shown here).

The main improvement that should be brought to the flash MMT method presented here is to design an injection system which will allow us to carry out a real flash water injection on the top surface of the fabric.

## 6. Conclusions

The flash MMT method presented in this work is based on the analogy with the flash method used in thermal engineering for the measurement of thermal diffusivities of materials. The 2D model developed simulates a water injection in the centre of the top surface of a fabric. The computation of the analytical solution of the resulting model is only possible if the initial condition is separable into a product of an axial term and a lateral term. The unknown model parameters, i.e. axial and lateral dispersion coefficients, were then identified from the available experimental measurements and their accuracy was quantified through confidence intervals.

The model predictions of WC were then compared to the experimental measurements at different levels: overall WC in the fabric and total WC at the top and bottom fabric surfaces. Although the results are quite interesting and encouraging, some improvements are still to be brought to the experimental measurement setup, mainly at the water injection level.

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