

## Optimisation based approach for designing flexible water usage network

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Key words: water usage network, design, uncertain data, flexibility, optimisation

Typical specifications for designing water usage network (WUN) involve mass load for every contaminant in each water using process. These data are highly uncertain and should be treated as random “disturbances” from given ranges. To account for the uncertainties a flexible WUN should be design such that meets the objectives of the minimum freshwater flow rate and the minimum number of connections for all possible mass loads. The design method for flexible WUN was developed. It is based on two major points: (i) design of optimal WUN-s for all corner points; (ii) embedding the solutions from step (i) into a flexible network by solving integer programming problem. The paper presents the method and an example of application.

### 1. Introduction

Due to increasing cost of water and stringent regulations on wastewater disposal to the environment a reduction of freshwater consumption as well as a decrease of wastewater generation by process integration is of great importance. The aims can be achieved in optimal water network, that uses minimum freshwater or is cost optimal in general. Numerous approaches for designing an optimal network consisting of water using processes, called water usage network (WUN) in the following, have been suggested in the literature. The approaches published to year 2000 were reviewed in Bagajewicz (2000), many new papers were published to date. Two types of water using processes were proposed: mass transfer operations and non-mass transfer processes known as sinks and sources of water. The former have generic model and, hence, are applied in this work. The typical data required to design WUN include mass loads of contaminants that have to be transferred in each water using process. They are highly uncertain in nature. Majority of designing approaches use fixed nominal values of these loads. Due to uncertain character of the parameters the solutions may be far from optimum or even they may be inoperable. Relatively few papers accounted for uncertain data. Suh and Lee (2002) are the authors of seminal work on WUN with considerations of uncertain data. They developed an approach for designing robust network. Koppol and Bagajewicz (2003) presented an approach for assessment of financial risk caused by uncertain data. To generate a flexible network they applied simultaneous method. The

method is limited to single contaminant case. Tan and Cruz (2004) suggested a design approach, also for single contaminant case, with uncertain mass loads (for mass transfer processes) or uncertain inlet concentration (for non-mass transfer processes). The uncertainties were handled by symmetric fuzzy linear programming model. Suad et al. (2005) developed an approach for multi-period WUN with regeneration. The design is performed in 3 stages: (i) network design for certain data, (ii) sensitivity analysis, (iii) stochastic optimisation for some seasonal scenarios. Foo et al. (2006) analysed influence of uncertain data on WUN design. They applied non-mass transfer model of water using processes. Tan et al. (2007) approached the problem of flexibility by simulation of WUN with fixed structure using mass loads generated by Monte Carlo generator from lognormal distribution. Liao et al. (2007) developed the synthesis approach of WUN for multiple plants and for multi-period scenario. The method was sequential with the use of optimisation. Finally, Shakhnovsky et al. (2007) dealt with data validation for WUN design by statistical analysis of measurements. Despite the efforts which shed some light on the complex problem there is no approach that can efficiently deal with design of flexible WUN for multiple contaminants.

In this contribution we will describe a systematic approach for designing flexible WUN. Such network ensures meeting two basic criteria of optimality commonly applied for WUN-s: the minimum freshwater consumption and the minimum number of connections (pipelines) for all values of mass loads of contaminants from given ranges.

## 2. Problem formulation

Given are water using processes ( $i=1, \dots, N$ ) where known contaminants are transferred to water. We assume that all processes can be modeled as counter-current mass transfer units. For every process  $i$  we know:

1. Maximum values of contaminants' concentrations at both inlet and outlet
2. Upper and lower bounds on mass loads of contaminants. It is assumed that the mass loads are random values of uniform distribution.

Furthermore we assume that there is one source of freshwater and the quality of this water is known, i.e. concentrations of the contaminants in freshwater are the data. No regeneration is applied in this network and no limitations are imposed on concentrations of wastewater streams leaving the network.

The objective is to design a flexible WUN. Flexible WUN, by definition, is such network that ensures the minimum freshwater flow rate and the minimum number of connections for each mass load value from the permissible region.

## 3. Foundations of the method

In the developed approach a flexible WUN is composed from component networks designed for fixed parameters from the data. The networks meet both optimisation criteria: the minimum freshwater flow rate and the minimum number of connections. The method is based on the assumption that the WUN is flexible if it meets the both criteria for certain data points called "corner" points. The corner points result from the "corner point theorem". They are combinations of both the maximum and the minimum contaminant mass loads for the processes. For the case of linear problem the corner theorem holds, i.e. the WUN flexible for the corner points is flexible in a whole space of

admissible disturbances. Additionally it is required that the minimum freshwater flow rate is used as the single performance index. WUN design problem with this criterion is linear if only one contaminant exists. To account for nonlinearities caused by multiple contaminants and, also, to take into consideration the second objective we add to the construction of the flexible structure certain networks that meet only one criterion - the minimum freshwater flow rate and have close to the minimum (but higher than that) number of connections. Finally, some other points are added during sequential solution procedure. Even though all these mechanisms do not guarantee the flexible WUN, the approach is expected to generate networks that are “sufficiently” flexible in practice.

The algorithm of the method is as follows:

- 1) Generate initial corner points.
- 2) Calculate optimal and near optimal networks for every point.
- 3) Calculate flexible WUN by solving integer programming problem.
- 4) Check flexibility of the WUN for some randomly generated data sets.
- 5) If the network failed the test for some points include them into the set of corner points and repeat calculations from step (2); otherwise stop calculations – the network is considered flexible one.

The steps of the algorithm will be explained briefly in the following section.

#### 4. Crucial steps of the method

The corner points are combinations of upper and lower limits on mass load of contaminants. The values of outlet concentrations are treated as certain data in this work. Generally, they can also be treated as uncertain parameters in frames of this approach. The illustration of corner points for uncertain mass loads is shown in the example in the next section.

To calculate optimal and near optimal WUN-s we applied the systematic approach by optimising superstructure with Adaptive Random Search (ARS) approach. The design method is described in Poplewski and Jeżowski (2005) in short and, also, in Jeżowski et al. (2007) in more detail. Stochastic optimisation methods are well suited for the purposes of directly generating some near-optimal networks. Note that after accomplishing step (2) we have some (let say  $m_i$ ) networks for each corner point ( $I$ ). The values of  $m_i$  are likely problem dependent and tests are carried out to find some rules on how to choose them.

To calculate flexible WUN we embed the “component” networks generated in step (2) into one frame – the flexible network. This would ensure that the final solution will meet the criterion of minimum freshwater flow rate. However, we should also minimize the number of connections within the flexible networks. Note also that the number of component networks can be large making a creation of the flexible solution by “hand” very difficult or even impossible. Hence, the use of optimisation is necessary. The problem of creating the flexible WUN from component networks was formulated as integer programming (IP) one. The objective is to minimize the number of all connections in the WUN. The connections include: pipelines from the freshwater source to the processes, from the processes to the disposal site and all connections amongst the processes. To define the component networks and the flexible WUN incidence matrix representation was used. The matrices for component networks are given in the data to

optimisation procedure. Each component network is uniquely defined by binary parameters such that:

$B_{ijkl} = 1$  if there is a connection among units  $i$  and  $j$  in network  $k$  of corner point  $l$ ; 0 – if not

The matrix for the feasible WUN is the result of optimisation and, thus, its entries are decision variables such that:

$Y_{ij} = 1$  if there is connection from unit  $i$  of the flexible WUN to unit  $j$ ; 0 – if not.

In order to ensure that the flexible WUN will use the minimum freshwater for all possible disturbances, at least one of the component network from a set generated for one corner point should be embedded into the solution. To code logical conditions additional binary variables were defined:

$Z_{kl} = 1$  if component network  $k$  for corner point  $l$  is in the solution; 0 – if not

Logical conditions that relate the variables  $Y$ ,  $Z$  and parameters  $B$  assure that variable  $Y_{ij}$  can take value of 1 if and only if binary  $Z_{kl}$  is equal to 1 and  $B_{ijkl}$  is 1 as well. Also, there are conditions that force the optimisation to include at least one component network  $k$  for corner point  $l$  into the feasible WUN. All the variables and parameters in the formulation of the optimisation problem are binaries. The goal function and all constraints are linear. Hence, there is linear IP formulation that can be solved to the global optimum using widely available optimisers.

The method of testing the flexibility is also based on optimisation. First, test data points are generated from the space of possible mass loads. For each point solutions of two WUN design problems are compared. First we solve the problem of determining the minimum freshwater consumption. Next, we solve the optimisation problem of minimization of the freshwater flow rate for the flexible network with valid upper limits on flow rates through connections. These upper bounds are calculated using known flow rates for component networks and binary variables  $Z_{kl}$  calculated in step (3). The equations act also as logical conditions setting flow rates for non-existing connections at zero. If for some test point there is a difference between the minimum flow rate for these two problems the conclusion has to be that the calculated WUN is not flexible. This test point is included into the set of corner points and the procedure starts again from point (2) of the general algorithm.

## 5. Example of application

Due to lack of literature examples for designing flexible WUN we adapted the example from Wang and Smith (1994) for single contaminant case. The data for four processes with the bounds on mass loads are shown in Table 1. Note that process no. 4 has fixed mass load. Table 2 shows corner points for this example with calculated minimum freshwater flow rate and number of solutions (optimum & near optimum networks) treated as component networks in the further steps of solution procedure. As shown in the last row of the table the number of connections in these networks ranges from 7 to 9. All the component networks feature the minimum freshwater flow rate. The flexible WUN is represented by Table 3 which shows the maximum flow rates via connections (empty entry means no connection). This network has 10 connections. Also, another solution has been calculated that has the same number of connections. It is shown in Table 4 in form of incidence matrix. Both solutions were tested according to step 4 of

the algorithm. Altogether 50 test points were used for each of them. Mass loads of contaminants were generated from the uniform distribution using Monte Carlo generator. Both networks passed the test and, thus, they were considered flexible ones.

Table 1 Data for the example

Process no.	$L^{min}$ [kg/h]	$L^{max}$ [kg/h]	$C_{in}^{max}$ [ppm]	$C_{out}^{max}$ [ppm]
1	2.0	2.2	0	100
2	5.0	5.1	50	100
3	30.0	30.5	50	800
4	4.0	4.0	400	800

Table 2 Corner points, minimum flow rates and solutions for the points

	Corner points								
L1	2.0	2.2	2.0	2.0	2.2	2.2	2.0	2.2	2.2
L2	5.0	5.0	5.1	5.0	5.1	5.0	5.1	5.1	5.1
L3	30.0	30.0	30.0	30.5	30.0	30.5	30.5	30.5	30.5
L4	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
Minimal total fresh water	90	92	91	90.333	93	92.333	91.333	93.333	93.333
Number of solutions	7	8	9	5	5	5	4	4	4
Number of connections in the solutions	7, 8	8	6, 7, 8	8, 9	7, 8	8	8	8	8

Table 3 Flexible WUN for the example (with values of maximum flow rates) – first solution

Inlet \ Outlet	P1	P2	P3	P4	Disposal site
P1		61.5266	40	22.0003	
P2			20.3334	52.9998	
P3					40.6672
P4					52.9998
Freshwater	88.5793	51	20.3334		

Table 4 Flexible WUN for the example (incidence matrix representation) – second solution

Inlet \ Outlet	P1	P2	P3	P4	Disposal site
P1		1	1		1
P2			1	1	
P3					1
P4					1
Freshwater	1	1	1		

## 6. Summary

The optimisation based approach has been developed for designing flexible network of water using processes. Some networks are calculated for corner points. The networks meet the criterion of minimum freshwater flow rate and have minimum, or close to the minimum, number of connections. Next, the feasible network is created by embedding them into the flexible structure. Integer programming problem is solved to minimize number of connection in this network. Finally, the flexible network is tested for some randomly generated points. The example of application is shown.

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