CHEMICAL ENGINEERING TRANSACTIONS Volume 17, 2009
Editor Sauro Pierucci

DOI: 10.3303/CET0917130

Effect of Joule Heating on Orientation of Spheroidal Particle in Alternating Electric Field

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We study the change of orientation of a spheroidal particle immersed into a host medium under the action of the alternating external electric field. It is assumed that the particle and the host medium have different electric conductivities. We show that the rate of Joule heating of the particle depends upon the orientation of its axis of symmetry with respect to the direction of the electric field. If electric conductivity of particle strongly varies with temperature, the Joule heating of the particle affects orientation dynamics and results in the appearance of the new equilibrium orientations. We investigate the dependence of the dynamics of particle rotation and the direction of the equilibrium orientation on the frequency of the electric field.

In our previous studies (Dolinsky and Elperin, 2005, 2006) we investigated the dynamics of a spheroidal particle (see Fig. 1) embedded in a leaky dielectric medium under the action of AC electric field $E_0(t) \approx E_0 \cos(\omega t)$.

We considered a particle with permittivity ε_2 and conductivity σ_2 that is embedded into a host medium with permittivity ε_1 and conductivity σ_1.

It was shown that stable equilibrium orientation (SEO) of the particle depends on the frequency ω of the applied electric field. In particular, under certain conditions which were established (Dolinsky and Elperin, 2006) there exist frequencies ω_1 and ω_2 such that in the frequency range $\omega_1 < \omega < \omega_2$ the direction of the axis of symmetry of the spheroidal particle is perpendicular to the direction of the same particle in an ideal dielectric system. For $\kappa_\varepsilon << 1$, where $\kappa_\varepsilon = \varepsilon_2/\varepsilon_1 - 1$, $\omega_2 \to \infty$ and the frequency ω_1 is determined by a simple formula:

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Please cite this article as: Dolinsky Y. and Elperin T., (2009), Effect of joule heating on orientation of spheroidal particle in alternating electric field, Chemical Engineering Transactions, 17, 777-782 DOI: 10.3303/CET0917130

$$\omega_1^2 = \frac{(1+f_z)(1+f_x)}{\tau_0^2}\ , \tag{1}$$

where $f_z = \kappa_\sigma n$, $f_x = \kappa_\sigma \dfrac{1-n}{2}$, $\kappa_\sigma = \sigma_2/\sigma_1 - 1$, $\tau_0 = \dfrac{\varepsilon_0 \varepsilon_1}{\sigma_1}$, and n are depolarization coefficients of a spheroid along the axis of symmetry.

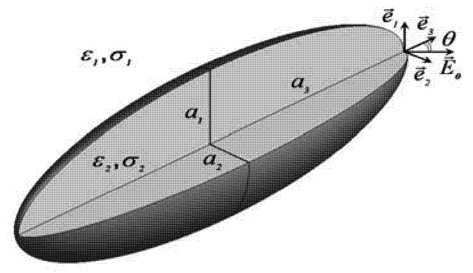

Fig.1. Ellipsoid with electric permittivity ε_2 and conductivity σ_2, inside a host medium with permittivity ε_1 and conductivity σ_1, in the external electric field $\vec{E}$.

The frequency threshold ω_1 separating the frequency domains with different orientation of the particle, depend upon the parameter κ_σ which depend upon the temperature of the particle and the host medium temperature. In the presence of Joule heating the frequency threshold ω_1 whereby the orientation of the particle flips to the perpendicular orientation, shifts. Since Joule heating rate depends on the orientation, in addition to the orientations with $\theta = 0$ and $\theta = \pi/2$, (θ – angle between the direction of AC electric field and axis of symmetry of spheroidal particle (see Fig.1) there appear new stable orientations of the particle. The main goal of this study is to elucidate the latter effect, namely, to determine stable orientations of the spheroidal particle in the presence of Joule heating.

Neglecting inertia, evolution of $\theta(t)$ in the presence of the electric field due the torque $\vec{M}(t)$ acting at the particle, is determined by the following equation:

$$\eta_s \frac{d\theta}{dt} = M(t), \tag{2}$$

where $\eta_s\, d\theta/dt$ is a torque acting at the particle due to the viscous force.

Neglecting spatial variation of temperature T_2 inside the particle, the energy balance equation can be written as follows:

$$\frac{dT_2}{dt} = \frac{T_1 - T_2}{\tau_T} + Q\,,$$
(3)

where τ_T is a characteristic temperature relaxation time which is assumed temperature independent and T_1 is external temperature. The thermal energy rate released by Joule heating per unit mass is given by the following formula:

$$Q = \left(\vec{j}_2 \cdot \vec{E}_2\right)\frac{1}{c\rho}\,. \qquad (4)$$

In the case of a strongly prolate spheroid $n \to 0$

$$\left\langle Q(t)\right\rangle = \frac{Q_0}{2}(1+\kappa_\sigma)\frac{2\left(1+\nu^2\right)+\kappa_\sigma+\kappa_\sigma^2/4+\cos(2\theta)\kappa_\sigma(1+\kappa_\sigma/4)}{\left(1+\nu^2\right)\left((1+\kappa_\sigma/2)^2+\nu^2\right)}\,, \qquad (5)$$

and

$$\left\langle M(t)\right\rangle = -\frac{M_0\kappa_\sigma^2\sin(2\theta)\left(1+\kappa_\sigma-\nu^2\right)}{\left(1+\nu^2\right)\left((1+\kappa_\sigma)^2+\nu^2\right)}\,, \qquad (6)$$

$$Q_0 = \sigma_1 E_0^2\left(1+\nu^2\right)/(4c\rho)\,,\ \nu = \omega\tau_0\,,\ M_0 = \varepsilon_0\varepsilon_1 V E_0^2/4\,. \qquad (7)$$

Let us consider now a simple but readily physically realizable particular case when a spheroid is a thin needle with sufficiently large electric conductivity so that the following conditions are met:

$$\kappa_\sigma \gg 1,\quad n \ll 1,\quad \kappa_\sigma n \ll 1\,. \qquad (8)$$

For future analysis it is convenient to present the temperature T in the form $T(t) = T_S\left(1 - \frac{E_0^2}{E_m^2 \alpha T_S} w(t)\right)$ where E_m, E_c are the characteristic values of the electric fields, $E_m^2 = 4c\rho(\tau_T |\chi_0|)^{-1}$, $E_c^2 = \eta\,(8\tau_T \varepsilon_0 \varepsilon_1)^{-1}$, and $\eta = \eta_S / V$ is the dynamic viscosity of the host fluid. The coefficient χ_0 in Eq. (6) is related with temperature coefficient of resistivity, α, by the formula $\chi_0 = -\alpha\sigma_2$, and χ_0 can be either positive or negative. The parameter T_S equals to the temperature of the spheroid when the angle between its axis of symmetry and the direction of the external electric field is $\theta = \pi/4$. Under these conditions Eq. (1), (2) can be rewritten as follows:

$$\frac{dw}{d\bar{t}} = -w + \mu, \qquad \mu = \cos(2\theta) \tag{9}$$

$$\frac{d\mu}{d\bar{t}} = sign(\chi_0)\frac{E_0^4}{E_m^2 E_c^2}\left(1-\mu^2\right)\left(w - sign(\chi_0)\frac{4E_m^2 \Delta}{E_0^2 \kappa_\sigma}\right) \tag{10}$$

The parameter $\Delta = v_S^2 - v^2$, where v_S^2 is determined by Eq. (1) for $T = T_S$.

For estimating the magnitude of E_m assume that $\alpha \approx 10^{-3}\,K^{-1}$, $c\rho \approx 10^6\ kJ \cdot kg^{-1} \cdot K^{-1}$ and electric conductivity $\sigma_2 \approx 1\,S \cdot m^{-1}$. These values of the parameters imply that $E_m \approx 10^5\ V/m$. The amplitude of the electric field E_c can be compared with E_c^Q, where E_c^Q is the magnitude of the electric field whereby the Quincke effect can be observed (Jones, 1995) in the fluid if the condition $\sigma_2 < \sigma_1$ is satisfied. This estimate is given by the following formula, $E_c / E_c^Q \approx \sqrt{\tau_0/\tau_T}$.

Numerical solutions of Eq. (8), (9) are showed in Fig. 2. Taking into account the mutual scale of variation of $\mu(t)$ and $w(t)$ we plotted these functions in the same figure. In this figure we denoted the numerically calculated plots of these functions by $X(t)$ whereby the curves with odd numbers correspond to the function $\mu(t)$ while the curves with even numbers correspond to the function $w(t)$.

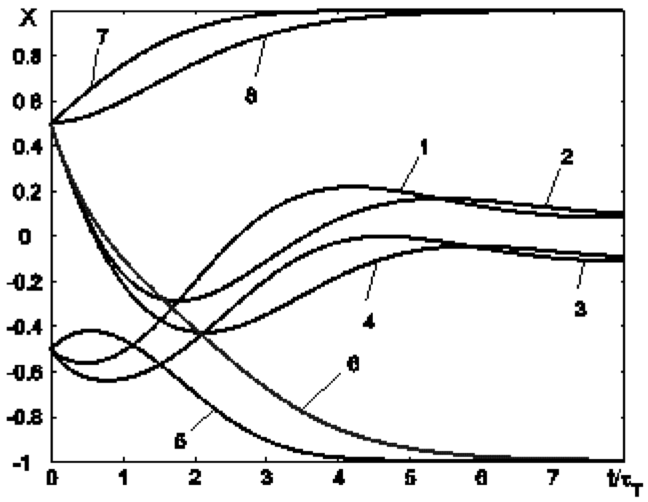

Fig. 2. Dependencies of the dynamic variables $\mu(t), w(t)$ *for various values of the parameters* $p_1 = sign(\alpha) E_0^4 / \left(E_m^2 E_c^2\right)$ *and* $p_2 = sign(\alpha) 4\Delta E_m^2 / E_0^2$ *. Curves 1, 2: 1-* $\mu(t)$*, 2-* $w(t)$*,* $p_1 = -1, p_2 = 0.1$*; Curves 3, 4: 3-* $\mu(t)$*, 4-* $w(t)$*,* $p_1 = -1, p_2 = -0.1$*; Curves 5, 6: 5-* $\mu(t)$*, 6-* $w(t)$*,* $p_1 = 1, p_2 = 0.1$*; Curves 7, 8: 7-* $\mu(t)$*, 8-* $w(t)$*,* $p_1 = 1, p_2 = 0.1$.

The time dependencies of the orientation angle $\theta(t) = \cos^{-1}(\mu(t))/2$ are showed in Fig.3.

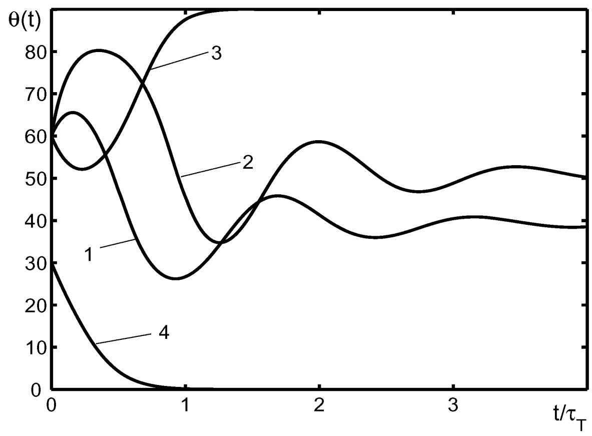

Fig. 3. Dependencies of the orientation angle $\theta(t)$ *for various values of the parameters* $p_1 = sign(\alpha) E_0^4 / \left(E_m^2 E_c^2\right)$ *and* $p_2 = sign(\alpha) 4\Delta E_m^2 / E_0^2$*. Curve 1:* $p_1 = -5, p_2 = 0.2$*; Curve 2:* $p_1 = -5, p_2 = -0.2$*; Curve 3:* $p_1 = 5, p_2 = -0.2$*. Curve 4:* $p_1 = 5, p_2 = 0.2$

Hence taking into account temperature dependence of the conductivity of a particle embedded into a host fluid qualitatively changes the dynamics of the system. The above presented analysis implies that depending on the sign of temperature coefficient of the resistivity α, the orientation of the embedded particle and the orientation relaxation behavior can be changed by varying the amplitude and frequency of the applied external electric field.

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