# Multi-Period Optimisation Of Dual-Mode Heat Pump Systems

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Multi-period mathematical programming approach is applied to enhance the design of dual-mode heat pump systems, which systematically consider variable ambient conditions and its influences to thermodynamic performance of heat pump. Natural working fluids are studied to provide sustainable solutions for residential heat pump applications. A system capacity of heat pump is optimised and economic trade-off between supplementary heating (or cooling) and operating cost is carried out. Case study is illustrated for demonstrating the benefits using the developed multi-period optimisation methods.

#### 1. Introduction

Heat pump systems have been widely regarded as a realistic and practical alternative to conventional mechanisms in the field of residential and industrial heating and cooling. Heat pump systems using natural external sources (e.g. air, seawater, etc) can provide much improved sustainability as less carbon emissions are generated by fully and strategically using environmental-benign resources. Also, sustainability in heat pump applications can be further improved by implementing natural working fluids to the system, rather than non-sustainable fluids (e.g. CFCs).

Both cooling in the summer and heating in the winter are required in many geographical areas, where a dual-mode heat pump system is often employed. With dual-mode operation for heat pump systems, heat is extracted from the heat source in the winter, and is circulated into the home, while, during the summer, heat is drawn from the interior air and rejected to the heat sink. Heat sources and sinks for residential heat pump applications can be ground surrounding a building, water in lakes, rivers or wells, and outdoor air. An air-source heat pump is considered in this paper as this is the most common type of heat pumps.

Climate conditions had been considered for the optimal design of reversible heat pump systems (Zhang et. al, 2007; Renedo et. al, 2006; Renodo et. al, 2007; Zogou and Stamatelos, 1998), but there still lacks systematic approaches which optimise dual-mode heat pump systems with considering simultaneously non-steady ambient conditions. A heat pump is often selected such that its output balances the heat requirement of the building in nominated ambient or design conditions (Electricity Council, 1978). This 'rule of thumb', however, does not guarantee a minimum cost or sustainable solution. Therefore, in this study, the integrated design of heat pump

systems has been studied to address seasonal variations of ambient conditions under dual-mode operation, in order to improve competitiveness of heat pump systems using environmental-friendly sources in the market, as well as to facilitate more active implementation of the heat pump in practice. Also, the current study focuses on the heat pump system using natural fluids and heating/cooling resources.

## 2. Multi-period Optimisation

The air-source heat pump system for residential heating and cooling normally includes compressor, condenser, evaporator, expansion valve and reversing valve. (Figure 1) Through different positions of reversing value, the heat pump system is operated in different modes: Indoor coil serves as a condenser in the heating mode, while the indoor coil is used as an evaporator in the cooling mode.

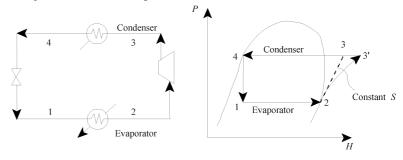


Fig. 1. Simple heat pump cycle and P-H diagram

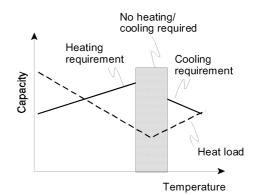


Fig 2. Variation of System Capacity

Given the geographical location and system on/off switch threshold, the heat pump is to be operated at different ambient temperature and consequently with different capacity and performance, as shown in Figures 2 and 3. If lower ambient temperature is chosen as the design condition, bigger capacity for heat pump system is required to meet higher heat demand, and less supplementary heating is necessary.

On the other hand, the design temperature becomes higher, shaftpower required for compressor becomes less while the need for supplementary heating becomes higher.

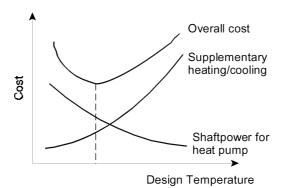


Fig 3. Economic trade-off for selection of design temperature

These two aspects will be systematically investigated through the developed optimisation model.

The mathematical model for the optimisation of heat pump systems is given as below. The required physical properties of refrigerants, i.e. enthalpy, density, and equilibrium pressure, can be calculated by the polynomial methods (Wang and Shan, 2001) regressed within the certain temperature range or the commonly used Equation of State (EOS) methods (Poling et. al., 2001).

Compressor (reciprocating):

$$W_{in} = \frac{m_r (h_3 - h_2)}{\eta_{isen}}$$
where  $\eta_{isen} = 0.109(\ln \gamma)^3 - 0.5247(\ln \gamma)^2 + 0.8577(\ln \gamma) + 0.3727$ 

$$m_r = V_P \eta_v \rho_{suc}, \text{ where } \eta_v = 98.6 - 5.6\gamma$$
(2)

Expansion (isenthalpic expansion is assumed.):

$$h_4 = h_1$$

System Capacity:

$$Q_c = Q_e + W = m_r (h_2 - h_1) + \frac{m_r (h_3 - h_2)}{\eta_{isen}}$$
(3)

Heat exchangers:

$$\begin{cases} Q_e = (UA)_e \Delta T_{emean} \\ Q_e = (FCp)_e \Delta t_e = (FCp)_e (T_{ein} - T_{eout}) \\ Q_e = m_r (h_2 - h_1) \end{cases}$$

$$(4)$$

$$\begin{cases} Q_e = (UA)_e \Delta T_{emean} \\ Q_e = (FCp)_e \Delta t_e = (FCp)_e (T_{ein} - T_{eout}) \\ Q_e = m_r (h_2 - h_1) \end{cases}$$

$$(5)$$

System efficiency (Coefficient of Performance):

$$COP_{cooling} = \frac{Q_e}{W}$$

$$COP_{heating} = \frac{Q_c}{W}$$
(6)

Heat load calculation

$$Q_{load} = 0.2 \times [23 - T_a] \tag{7}$$

Model constraints:

$$\begin{cases} T_{cin}(i) = T_a(i) \\ T_{ein}(i) = T_a(i) \\ IH(i) = 1 \quad if \quad T_a(i) \le 15 \\ IC(i) = 1 \quad if \quad T_a(i) \ge 25 \\ IH(i) + IC(i) \le 1 \end{cases}$$

$$(8)$$

Objective function:

min 
$$Cost = \sum_{i} W(i) \times D(i) \times P + \sum_{i} IH(i) \times (Q_{Load}(i) - Q_{c}(i)) \times D(i) \times P_{SH} + \sum_{i} IC(i) \times (Q_{Load}(i) - Q_{e}(i)) \times D(i) \times P_{SH}$$

$$(9)$$

### 3. Case Study

Ambient conditions determine the operating mode, operating conditions and operating durations of a dual-mode heat pump system. Ambient conditions depend on the geographical location.

Normally, monthly average temperature, average high temperature and average low temperature are available Simple relationship is assumed to calculate the heat load (Canadian Building Digests, 2007). The on/off switch threshold for heating is 15°C and for cooling is 25°C. In this way, a whole year will be broken into multi-periods, which

include the information of ambient temperature, cooling or heating mode, and duration for each operating period. Figure 4 shows climate conditions and its multi-period representation of heating and cooling demand for case study.

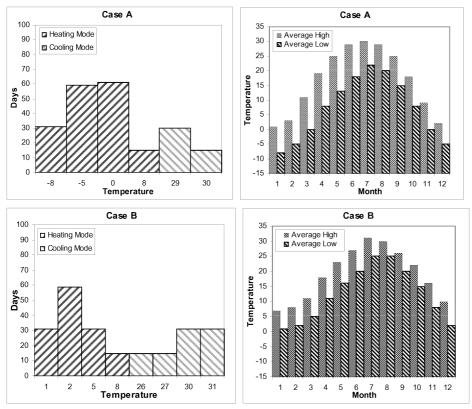


Fig. 4 Climate conditions and discrete multi-period conditions

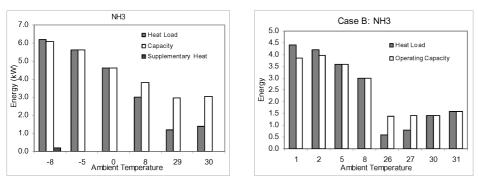


Fig 5. Heat load and capacity at the optimal design using ammonia

In Case A, the heating requirement in the winter is more than cooling requirement in the summer. Case B needs relatively equal heating in the winter and cooling in the summer, as the ambient temperatures in the winter are mild compared to Case A. The optimal results for both cases are given in Figure 5. For Case A, the supplementary heating is

introduced to avoid excessive capacity (i.e. size) of heat pumps, while no supplementary heating is required for Case B. It should be noted that the optimal capacity of heat pump systems is strongly dependent on ambient conditions as well as economic scenarios (i.e. supplementary energy costs vs. power cost for heat pumps).

## 4. Summary

In this study, the integrated design and optimisation method for heat pump systems are proposed to improve cost-effectiveness and thermodynamic efficiency of heat pump systems using natural fluids.

The multi-period formulation technique is used to consider variable ambient conditions for air-source dual-mode heat pump systems. The developed methodology can be used as a decision-support for the design of residential heat pump system using natural working fluids.

#### Acknowledgement

Authors would like to acknowledge the financial and informational support from EC FP6 Collective Research Project SHERHPA COLL-CT-2004-500299.

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