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Optimal Batching and Scheduling of Single Stage Batch Plants with Sequence Dependent Changeovers

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This paper presents a new multiple-time grid, continuous-time, mixed integer linear programming (MILP) model for the optimal short-term scheduling of single stage batch plants and optimal selection of the number of batches to produce. It considers aggregated processing and changeover tasks that account for the time required to produce all batches of the product, plus the changeover time between the different batches, plus the required changeover to the next product in the sequence. When compared to the traditional approach of considering a single combined processing and changeover task per batch, the new approach is significantly more efficient as illustrated through the solution of 12 example problems for sales revenue maximization.

1. Introduction

Optimization models for batch scheduling can be classified according to four main features (Mendéz et al., 2006). One refers to the material balances and the way in which batches and batch sizes are handled. Models based on unified frameworks like the State Task Network (STN) or the Resource Task Network (RTN) can simultaneously deal with the optimal set of batches (number and size), the allocation and sequencing of manufacturing resources, and the timing of processing tasks. Alternatively, we have models that assume that the number of batches of each size is known in advance, which can be regarded as one of the modules of a solution approach for detailed production scheduling, widely used in industry, which decomposes the whole problem into two stages, batching and batch scheduling.

This paper builds on recent work by the authors (Castro et al., 2006), who have compared different solution procedures for the batch scheduling of multistage batch plants with sequence dependent changeovers. Now, however, instead of considering that the number of batches for the production of a given order is known in advance, we solve the simultaneous batching and scheduling problem. Two RTN-based multiple time grid continuous-time formulations are considered that handle the batching problem differently. One is the standard approach where the number of batches is determined implicitly by the number of processing tasks that are needed in the optimal schedule and each task corresponds to a single batch. The second and new approach, considers aggregated tasks that include all the required batches of a certain order, where only one will need to be executed in a particular equipment unit. The number of batches is considered explicitly as a model variable, which is multiplied by the duration of a batch to determine the full length of the aggregated task. In this way, we take advantage of the ability of continuous-time formulations to handle variable duration tasks.

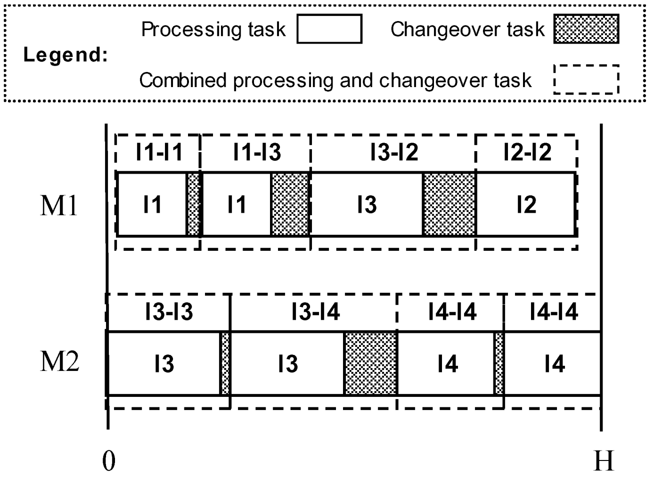

Figure 1. Implicit batching approach

2. Problem Statement

We consider the optimal short-term scheduling of single stage batch plants together with the selection of the optimal number of batches. Given are a set *I* of products to be produced in a set *M* of available equipment units. Both the duration $p_{i,m}$ and batch size $b_{i,m}$ of product *i* in unit *m* are known and assumed to be fixed. Given also are the duration of the required changeover times between the products, $cl_{i,i',m}$, and the product demand, Δ_i. The objective function will be the maximization of the sales revenue over a fixed time horizon, *H*, where the product value is given by v_i. Note that the specified demand will typically not be met for all the products.

3. Implicit Batching Approach (IB)

The implicit batching approach considers the traditional definition of a processing task, the activity to process one batch of a particular product. Like in STN/RTN models, it implicitly decides the number of batches to produce in the given time horizon by the number of task instances that are executed. Figure 1 illustrates IB with a simple example, where 2 batches of product I1, 1 of I2, 3 of I3 and 2 of I4 are produced. Note that in order to allow for maximum plant flexibility we do not restrict all batches of a single product to be produced in a single equipment unit (e.g. I3). Also, we may have nonzero changeovers between different batches of the same product.

The implicit batching approach relies on the RTN process representation and employs multiple time grids, one for each equipment unit. It uses four-index binary variables linked to the execution of combined processing and changeover tasks like in formulation CT4I of Castro et al., 2006. While that formulation assumed that the amount of material processed was implicit on the task duration, IB has to deal with material amounts and eventually with variable duration tasks, so we have used insights from the single time grid formulation of Castro et al., 2004.

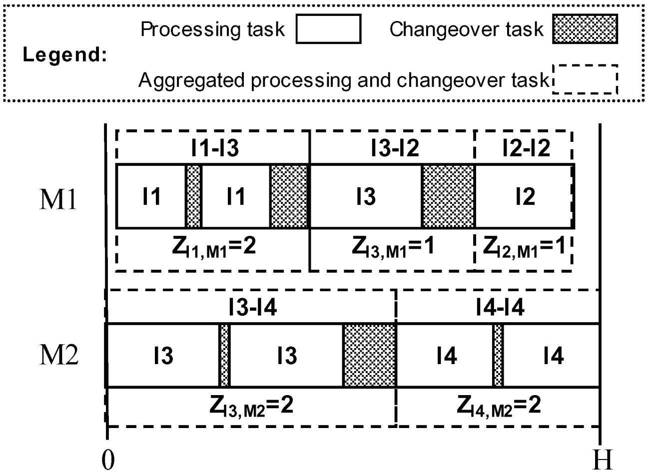

Figure 2. Explicit batching approach

4. Explicit Batching Approach (EB)

The explicit batching approach takes advantage of the fact that time grid based continuous-time formulations can handle variable duration tasks without significantly altering the complexity of the model. All processing instances of the same product that are executed in the same unit are considered as a single aggregated task, with the integer variable $Z_{i,m}$ defining the number of batches to produce. In EB, the aggregated task linked to product i accounts for the aggregated processing time, the changeover time between different batches of the same product (as many times as the number of batches allocated to the unit minus one) and the final changeover time that prepares the unit for the subsequent product, see figure 2.

The explicit batching approach requires two sets of extent variables to characterize an aggregated task. Binary variables $N_{i,i',m,t}$ identify the execution at event point t of the task required to produce product i in unit m followed by the changeover task to product i', while the positive continuous variables $\xi_{i,m,t}$ give the amount of i processed in unit m at time t. The initial condition of unit m is determined through binary variables $C^0_{i,m}$, which can be fixed if the initial state is known. The excess resource variables $C_{i,m,t}$ indicate the availability of unit m at a condition that enables it to process i at event point t. While these variables could be defined as binary, it is not necessary since the model constraints ensure that $C_{i,m,t}$ ={0,1}. It is also important to note that the sum over i is linked to the availability of the unit, for which variables $R_{m,t}$ could be used explicitly (Castro et al., 2006). Finally, $T_{t,m}$ represents the absolute time of event point t belonging to time grid m.

The domain of variables $N_{i,i',m,t}$ is defined through set $I_{i',m,t}$ (eq 1), which accounts for the fact that tasks with different order indices cannot be executed in the last time slot, while those with the same order index are restricted to the last interval. Set I_m includes the orders that can be processed in unit m, those that have a nonzero duration (eq 2). The model constraints start with the excess resource balances, which can be viewed as multiperiod material balance expressions where the excess amount at point t is equal to that at point t-1 adjusted by the amounts produced/consumed by all tasks starting or ending at t. In eq 3, condition i is produced by all tasks (i',i) starting at t-1 and is consumed by tasks (i,i') starting at t. Eq 4 ensures that there is but one initial condition for each unit. There is exactly one equipment unit of type m, so the maximum availability at any time point is equal to one, eq 5. At most one aggregated task of product i will be executed in unit m since one task can handle multiple batches. The amount of i produced in unit m must be equal to the batch size times the number of batches of that product in that unit, eq 7. Eq 8 places an upper bound on the integer variables. The ceiling function has been used due to the fact that the production of i cannot exceed its demand (eq 9). Product i can only be produced if one of its aggregated tasks is active at that point. The binary and continuous extent variables are related through eq 10, where the upper bound on the amount produced is the product demand and the lower bound is the batch size. Eq 11 defines the essential timing constraints, the heart of continuous-time formulations. For each aggregated task we need to account for the total processing tasks, total changeover time between different batches of the same product and changeover time for the following product, the latter only for tasks not executed in the last time interval (see figure 3). Eq 12 sets the time of the first event point to zero, for all time grids, while the time horizon acts as the upper bound for all points (eq 13). Finally, eq 14 defines the objective function, the maximization of sales revenue. Operating and changeover costs can easily be incorporated, the latter in a way similar to that used to determine the duration of the task (see eq 11 and figure 3) .

$$I_{i',m,t} = \{i \in I_m : (t \neq |T|-1 \wedge i \neq i') \vee (t = |T|-1 \wedge i = i')\} \ \forall m \in M, i' \in I_m, t \in T, t \neq |T| \quad (1)$$

$$I_m = \{i \in I : p_{i,m} > 0\} \ \forall m \in M \quad (2)$$

$$C_{i,m,t} = C^0_{i,m}\Big|_{t=1} + C_{i,m,t-1}\Big|_{t\neq 1} + \sum_{i' \in I_{i,m,t-1}} N_{i',i,m,t-1} - \sum_{\substack{i' \in I_m \\ i \in I_{i',m,t}}} N_{i,i',m,t} \ \forall m \in M, i \in I_m, t \in T \quad (3)$$

$$\sum_{i \in I_m} C^0_{i,m} = 1 \ \forall m \in M \ ; \ C_{i,m,t} \leq 1 \ \forall m \in M, i \in I_m, t \in T \quad (4\text{-}5)$$

$$\sum_{t \in T} \sum_{\substack{i' \in I_m \\ i \in I_{i',m,t}}} N_{i,i',m,t} \leq 1 \ \forall m \in M, i \in I_m \quad (6)$$

$$\sum_{t \in T, t \neq |T|} \xi_{i,m,t} = b_{i,m} \cdot Z_{i,m} \ \forall m \in M, i \in I_m \ ; \ \overline{Z}_{i,m} = \lfloor \Delta_i / b_{i,m} \rfloor \forall m \in M, i \in I_m \quad (7\text{-}8)$$

$$\sum_{\substack{t \in T \\ t \neq |T|}} \sum_{\substack{m \in M \\ i \in I_m}} \xi_{i,m,t} \leq \Delta_i \ \forall i \in I \quad (9)$$

$$b_{i,m} \cdot \sum_{i' \in I_m, i \in I_{i',m,t}} N_{i,i',m,t} \leq \xi_{i,m,t} \leq \Delta_i \cdot \sum_{i' \in I_m, i \in I_{i',m,t}} N_{i,i',m,t} \ \forall m \in M, i \in I_m, t \in T, t \neq |T| \quad (10)$$

$$T_{t+1,m} - T_{t,m} \geq \sum_{i \in I_m} \left[\frac{\xi_{i,m,t}}{b_{i,m}} \cdot (p_{i,m} + cl_{i,i,m}) + \sum_{\substack{i' \in I_m \\ i \in I_{i',m,t}}} N_{i,i',m,t} \cdot (cl_{i,i',m}\Big|_{t \neq |T|-1} - cl_{i,i,m}) \right] \forall m, t \neq |T| \quad (11)$$

$$T_{1,m} = 0 \; \forall m \in M \; ; \; T_{t,m} \leq H \; \forall m \in M, t \in T \qquad (12\text{-}13)$$

$$\max \sum_{\substack{t \in T \\ t \neq |T|}} \sum_{m \in M} \sum_{i \in I_m} v_i \cdot \xi_{i,m,t} \qquad (14)$$

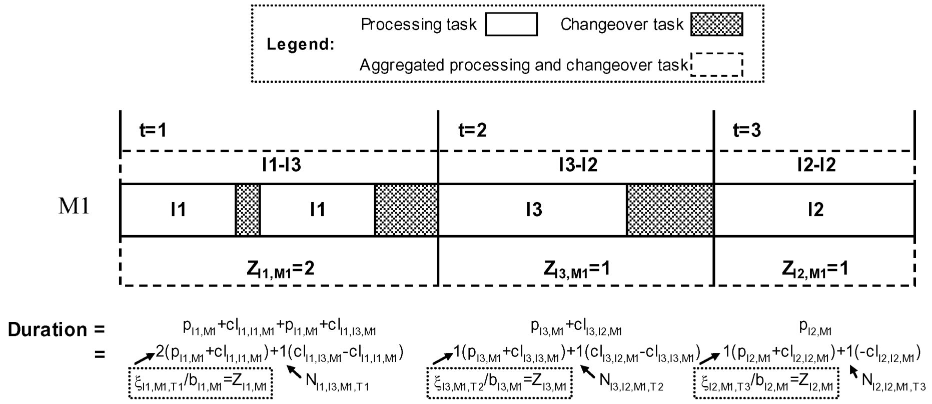

Figure 3. Calculation of the duration of the aggregated tasks

5. Computational Studies

The performance of the two approaches is evaluated through the solution of 4 example problems, each solved for three different values of the time horizon, *H*. Naturally, the higher the time horizon the higher the revenue, due to the production of a higher number of batches. The problem data was taken from a real industrial plant. The continuous-time mathematical formulations were implemented and solved in GAMS 22.2 using CPLEX 10.0.1 as the MILP solver. All the problems were solved to optimality (relative tolerance=1E-6) or up to a maximum resource limit of 1 h. The computer used was a Pentium-4 3.4 GHz processor with 2 GB of RAM, running Windows XP Professional.

Table 1 shows the global optimal solution, computational effort and the number of event points used to solve each problem. Note that although all problems were solved to optimality, it cannot be ensured that the values reported for two instances of P4 are the global optimal solutions since the number of event points employed can act as a constraint on the feasible region and we were unable to solve the problem for another increment in |T|. The results undoubtedly show that the explicit batching (EB) approach is the best one. The implicit batching approach (IB) is a worse performer by typically one order of magnitude, an expected behaviour since it requires a larger number of discrete variables and, most of the times, also exhibits a larger integrality gap (such results are not given). Contributing to the number of discrete variables are the number of binary and integer variables. EB uses the exactly same set of binary variables ($N_{i,i',m,t}$ being the most important) as IB plus the integer variables $Z_{i,m}$. However, the number of binary variables in IB is significantly higher due to the need to use time grids with more event points (i.e. index *t* has a wider range) to find the exact same solution as EB.

Table 1. Computational statistics

problem	H (hrs)	optimum	Effort (CPUs)		Event points (|T|)	
			IB	EB	IB	EB
P1 (|I|=5, |M|=2)	120	908.7	2.95	1.19	9	4
	144	1036.5	6.17	0.84	10	4
	168	1149.3	4.72	0.55	11	4
P2 (|I|=8, |M|=2)	144	1202.8	18.2	1.5	11	5
	168	1329.2	465	1.73	13	5
	192	1464.4	48.5	0.52	14	5
P3 (|I|=10, |M|=2)	120	1218.5	723	285	11	7
	144	1388.88	853	49.1	12	7
	168	1544.88	14386	0.19	13	7
P4 (|I|=10, |M|=4)	96	1742.14[a]	3600[b]	2292	7[e]	5
	120	2068.58[a]	3600[c]	14669	8[e]	5
	144	2196.68	3600[d]	79.4	8[e]	4

[a]Global optimal solution is still unknown; [b]Maximum resource limit (MRL), suboptimal solution (SO)=1751.3; [c]MRL, SO=2028; [d]MRL, SO=2183.1; [e]More event points are required to find the global optimal solution.

6. Conclusions

This paper has presented a new continuous-time formulation for the simultaneous batching and scheduling of single stage batch plants in which equipment units are subject to sequence dependent changeovers. The formulation relies on the use of multiple time grids, one per equipment resource, and uses a set of integer variables to explicitly determine the number of batches of each product to produce in a particular unit. In this way, all individual processing tasks (one per batch) can be included in a single aggregated task per product and fewer event points need to be used to solve the problem to global optimality, which is translated into a lower computational effort.

Work is currently in progress to compare the new formulation to one using global precedence sequencing variables (Harjunkoski & Grossmann, 2002) and to other relying on immediate precedence sequencing constraints that can be viewed as a relaxation of the travelling salesman problem (Erdirik-Dogan & Grossmann, 2007).

7. References

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