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Operation Planning Under Product Demand Uncertainty In Complex Chemical Plants

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This work deals with the operational planning and process optimization under uncertainty for ethylene plants through the formulation of a two-stage stochastic model which is transformed into a deterministic mixed integer nonlinear (MINLP) one. The model includes mass balances and nonlinear correlations for the furnaces and the entire plant. Decay in furnaces performance throughout operating time has been modelled by means of two sets of binary variables and several continuous time varying variables. First stage decisions are associated to manufacturing variables, while second stage decisions are related to logistics. The model comprises more than 10000 constraints and 256 binary variables and the resulting MINLP problem has been solved in GAMS using DICOPT++.

1. Introduction

In the recent years, there has been an increased interest in planning and scheduling under uncertainty. Demand uncertainties have received special attention, since in nowadays´ competitive and changing environments, planning output levels is crucial for surviving in the business. In this paper, optimal scheduling and process optimization under uncertainty in an ethylene plant has been considered through the formulation of a two-stage stochastic model which is transformed into a deterministic mixed integer nonlinear (MINLP) one. The decay in furnaces performance throughout operating time has been modelled by means of two sets of binary variables and several continuous time varying variables (Schulz *et al.*, 2006a). The main one is coils' roughness, an empirical linearly increasing continuous variable in each time period, whose dependence on time has been determined through rigorous simulations and checked with plant data. There are first stage decisions associated to manufacturing variables that include production levels, process units operating conditions and furnaces run lengths. Second stage decisions are related to logistics; they include inventory levels, product sales, shortage of product and deviation from target inventory levels. The objective function is to minimize expected cost and it is composed of two terms. The first term captures the costs associated to the manufacturing phase, i.e., the sum of the first-stage costs, which are deterministic and include raw material and production costs. The second term comprises the expected value of the second-stage costs, which quantify the costs

associated to inventory holding charges, safety stock violation penalties and penalties for lost sales. They are related to logistics decisions. This second term is obtained by applying the expectation operator to an embedded optimization problem.
The solution of the two-stage stochastic problem is obtained by solving its deterministic equivalent problem, assuming that product demands are distributed on a discrete probability space. The resulting reformulated MINLP problem has been solved in GAMS (Brooke *et al.*, 1992) using DICOPT++ (Viswanathan and Grossmann, 1990).

2. Mathematical model

A typical ethylene plant consists of several parallel pyrolysis furnaces, a cracked gas compressor, heat recovery network, separation system, refrigeration system and steam plant. The plant fresh feed, mainly ethane, bends with an ethane recycle stream and ethane from a storage and is then diluted with process steam to diminish coke deposition. Thermal cracking in the furnaces produces ethylene and subproducts. The outlet gas is cooled and compressed to cryogenic conditions and is afterwards fractionated in the separation train: demethanizer, deethanizer, depropanizer, debutanizer and two splitters to further separate ethane from ethylene and propane from propylene.
The proposed stochastic multiperiod model considers uncertainty in demands while avoiding overproduction, which leads to high inventory costs, and shortfalls (missed sales). Cyclic scheduling of shutdowns for eight parallel ethane cracking furnaces is modeled as a multiperiod Mixed Integer Nonlinear Programming (MINLP) problem, with discrete time representation. A fixed cycle length, based on plant historical data, has been considered for all furnaces.
The objective function, Equation (1), is the maximization of the expected profit.

$$max\ E\left[\sum_t \sum_i RE\left(\theta_{i,t}\right) + IC\left(\theta_{i,t}\right) + UP\left(\theta_{i,t}\right)\right] - \sum_t \sum_i PC_{i,t} \qquad (1)$$

where θ is the stochastic parameter vector of demands, E stands for expected value, RE for revenues, IC for inventory costs, UP for penalty for underproductions (unmet demand) and PC for production costs (heating, cleaning, inventory and raw material costs, and penalties for unmet security level in ethane storage tank), for products i and time periods t. The objective function is subject to furnaces shutdown scheduling constraints, production correlations and mass balances on equipment and storage tanks.
An equivalent deterministic mixed integer nonlinear (MINLP) problem is derived. Demand uncertainty is considered as a set of three scenarios, each representing a possible shifting of market expectations. Demand probabilities have been discretized with probabilities *probp* (*probp* = 0.31, 0.38, 0.31, for the respective scenarios *p*, below-average, average and above-average). The objective function is now expressed as Equation (2):

$$\begin{aligned} max \sum_t \sum_i \sum_p\ prob_p * \Big(sale_{i,t,p} * price_i + delta_{i,t,p} * penalty_i + \\ area_{i,t,p} * invcost_i\Big) - \sum_t \sum_i PC_{i,t} \end{aligned} \qquad (2)$$

where the first term corresponds to the revenues and the second one is the penalty for the unmet demands. The third term is the inventory cost which is, for each period, proportional to the trapezoidal area under the inventory function ($area_{i,t,p}$).

Furnace Shutdown Scheduling. Binary variables $z_{h,t}$ are defined by Equations (3) and (4), where TP_h is the cleanup period for furnace h. If period t is before or equal to TP_h, $z_{h,t}$ is 0, otherwise $z_{h,t}$ equals 1.

$$t \le TP_h + BM1 * z_{h,t} \qquad \forall h,t \tag{3}$$

$$t \ge (TP_h + 1) - BM1 * (1 - z_{h,t}) \qquad \forall h,t \tag{4}$$

Furnaces decaying performance is described through coil internal roughness. This empirical variable has been correlated based on rigorous simulations of the plant and it has a linear dependence on furnaces operating time. The following Big-M formulations, Equations (5) to (8), model roughness, $Rug_{h,t}$, behavior for each furnace h at time period t. Coil roughness increases linearly with operation time and lowers to a minimum roughness value ($C1clean_h$ = 6.42 E -4) when the furnace is cleaned. After the shutdown period, roughness increases linearly again. All furnaces have different initial conditions (different parameters $C1_h$) and different slopes in the roughness correlations (different parameters $C2_h$).

$$Rug_{h,t} \ge C1_h + C2_h * t - BM2 * z_{h,t} \qquad \forall h,t \tag{5}$$

$$Rug_{h,t} \le C1_h + C2_h * t + BM2 * z_{h,t} \qquad \forall h,t \tag{6}$$

$$Rug_{h,t} \ge C1clean_h + C2_h * \left[t - (TP_h + 1) \right] - BM2 * (1 - z_{h,t}) \qquad \forall h,t \tag{7}$$

$$Rug_{h,t} \le C1clean_h + C2_h * \left[t - (TP_h + 1) \right] + BM2 * (1 - z_{h,t}) \qquad \forall h,t \tag{8}$$

The heat load to furnace h, $Qf_{h,t}$, depends not only on the furnace load but in the operation time (it also decreases after the furnace has been cleaned and starts increasing as operation begins), therefore, it has been modeled with analogous inequalities.

There is a second set of binary variables associated to furnaces shutdown, $y_{h,t}$, which is 1 when furnace h is shutdown in time period t and 0, otherwise. Equation (9) defines the shutdown period for furnace h, TP_h, and Equation (10) states that each furnace can be cleaned only once throughout the time horizon.

$$TP_h = \sum_t t \; y_{h,t} \qquad \forall h \tag{9}$$

$$\sum_t y_{h,t} = 1 \qquad \forall h \tag{10}$$

At most, two furnaces can be simultaneously shutdown during each time period. A fixed cycle time of 16 weeks and a shutdown period of one week are considered for all furnaces. The cyclic nature of the problem is modeled by imposing that final conditions in coil roughness and inventory levels must be the same as initial ones.

Production Correlations. The entire plant model downstream furnaces is included to capture the influence of an important ethane recycle, which is part of the furnaces feed. Big M inequalities have been included to ensure that the feed and production of each furnace are zero when it is being cleaned. A detailed description of the ethylene plant model can be found in Schulz *et al.* (2006a).

Mass Balances on Storage Tanks. The mass of product i stored in period t, $V_{i,t,p}$, is the mass at the beginning of the horizon, $V_{i,0}$, plus what has been produced of i minus what has been sold until period t.

$$V_{i,t,p} = V_{i,0} + \sum_{1}^{t} production_{i,t} - \sum_{1}^{t} sale_{i,t,p} \qquad \forall i,t,p \qquad (17)$$

Mass balance equations have been also defined for ethane storage, and an economic penalty has been included in the objective function when the stored ethane is above or below a security level.

Penalty for Unmet Demands. Equation (18) states that sales ($sale_{i,t,p}$) cannot exceed the forecast demand ($dem_{i,t,p}$). Furthermore, the unmet demand is monitored by a penalty included in the objective function.

$$sale_{i,t,p} \leq dem_{i,t,p} \qquad \forall i,t,p \qquad (18)$$

$$sale_{i,t,p} + delta_{i,t,p} = dem_{i,t,p} \qquad \forall i,t,p \qquad (19)$$

3. Numerical Results

In the present analysis, uncertainty has been considered for the demands of the two most important products, ethylene and propane. The model provides a cyclic schedule for the cleanup shutdowns for the eight furnaces on a fixed cycle of 16 weeks. Roughness follows a linear behaviour, increasing until the cleanup period (TP_h) when it decreases to a minimum value (*C1clean*) and after furnace cleaning, it increases again. The model contains 10283 constraints, 5369 continuous variables and 256 binary variables. The problem has been solved in GAMS with DICOPT++ in four major iterations, with CONOPT3 and CPLEX, in 4610 sec. in a Pentium IV.

Numerical results obtained with the proposed stochastic model have been compared to those from the deterministic model (Schulz et al., 2006). The latter considers ethylene and propane demands 10% above the corresponding mean value. This fact explains the fact that the overall ethylene production, which is a first-stage decision variable, is higher in the deterministic case (Figure 1). In Figure 2, ethylene sales for the three scenarios are shown. Table 1 shows the shutdown periods for the deterministic and the stochastic cases. It can be seen that the optimal shutdown schedules follow similar trends.

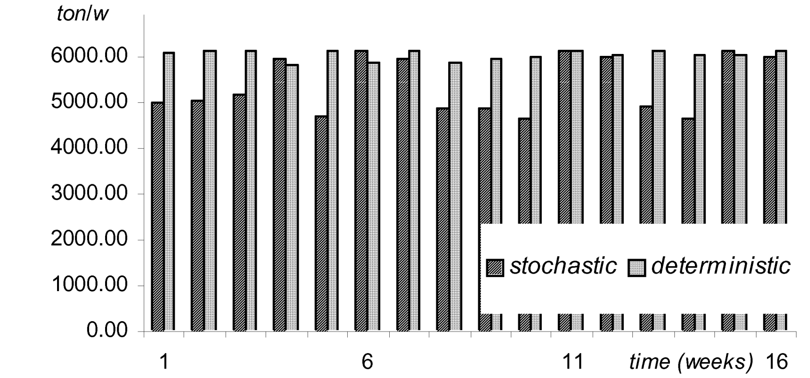

Figure 1. Ethylene production for the deterministic and stochastic cases

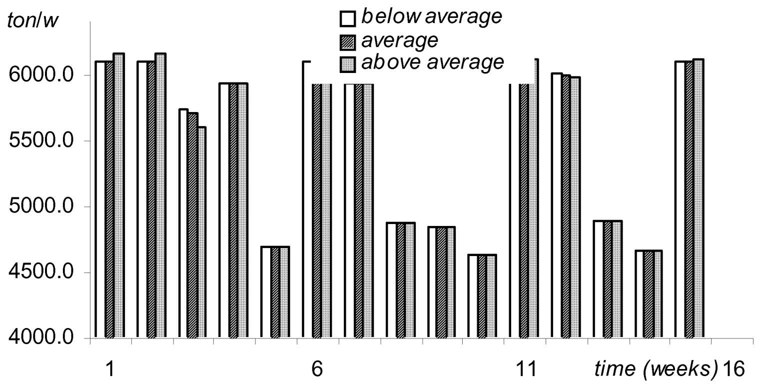

Figure 2. Ethylene sales for the three scenarios

Table 1. Shutdown periods

Furnace	Deterministic	Stochastic
1	15	14
2	12	13
3	9	9
4	8	10
5	6	8
6	4	5
7	14	14
8	10	10

Figures 3 shows propane production, sales, demand and storage profiles for the below-average scenario. Production is a first stage variable, therefore it is the same for all the scenarios. In the below-average scenario, the demand is satisfied in all the periods and there is an overproduction that is stored. However, when demands are higher in the average scenario, the production is not enough to completely satisfy the demands in all the periods even making use of the stored propane. Therefore, in the last period the production has to be used to satisfy the cyclic inventory constraints.

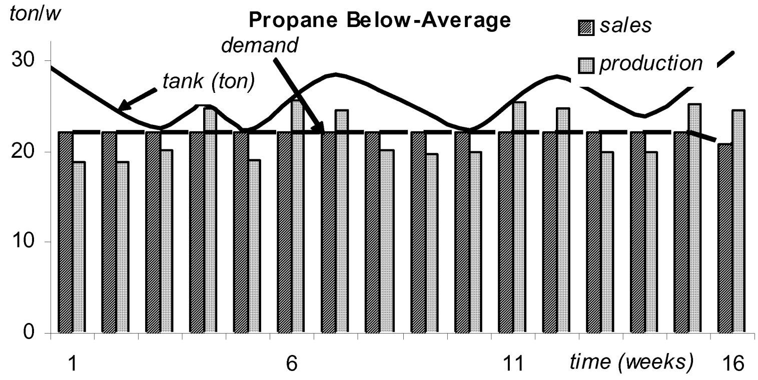

Figure 3. Propane production, sales, demand and storage for below-average scenario

4. Conclusions

Optimization results show that market demands are key parameters that have strong effects on plant operation planning. As first and second stage decision variables are different from the deterministic ones, it can be concluded that deterministic optimization models can result in suboptimal planning decisions when uncertainty in market demands is overlooked.

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