# Robust Decentralized Control For Flexible Plant Operation: Conceptual Design And Optimal Tuning

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In this work we describe a conceptual approach which has been recently developed to design efficient decentralized control structures for flexible chemical plant operation. The approach combines concepts from thermodynamics, inventory networks and process control to construct a set of stable and robust decentralized control candidate structures. Such candidates consist of inventory control loops designed so to ensure convergence of the mass and energy inventories to given references and thus guaranteeing stability of the extensive properties despite plant disturbances or even parameter uncertainty. Since the loops are defined over inventories, they contain almost every possible decentralized alternative. In this way, and by physically realizing the inventory control loops over the available degrees of freedom, the approach can be employed to construct a superstructure only containing stable (and robust) decentralized control candidates. It is on this set that NLP or MINLP problems can be defined to select the best operational alternative. The different aspects of the conceptual design approach will be illustrated on the well known Tennessee-Eastman Process. In particular the benchmark case will be employed to illustrate the definition of the stable inventory alternatives, physical realization and optimal selection and tuning by an efficient and recently developed MINLP solver.

#### 1. Introduction

Over the years, the area of decentralized plant-wide control has attracted the process engineering community as a challenging problem which drives continuing research efforts. The *thermodynamic-based control* (TBC) design procedure developed by Antelo *et al.* (2006, 2007) tried to provide a systematic to this framework of the plant-wide control. The basic ingredients of the theory have been established by Alonso and Ydstie (1996) and Ydstie and Alonso (1997) in the context of passive control design and by Alonso and Ydstie (2001) for the control of distributed systems. A similar line of arguments was employed by Farschman *et al.* (1998) to derive mass and energy inventory control concepts. Finally, Antelo *et al.* (2006, 2007) set up the basis for exploiting the underlying algebraic structure of process networks to define a decomposition of fundamental process networks into mass and energy inventory layers, over which conceptual inventory control loops can be defined in more clear way.

A key step in this TBC methodology is the formalism concerning the *conceptual inventory control design*, which ensures the convergence of the process states to compact regions of the state space constrained by constant total mass and energy. In these sets, thermodynamics gives us a function -the entropy of the system- which has a definite curvature (concavity - Ydstie and Alonso, 1996). Moreover, the function has a well-defined maximum in those regions. Such function will be the one employed to

derive natural storage and Lyapunov function candidates of use in designing robust controllers as presented in the work by Alonso and Ydstie (2001). These conceptual inventory control loops consist of linear proportional-integral controllers for mass and energy inventories, which have to be translated into suitable control loops over the real manipulated variables available in the process. This procedure is the so called *control* loop realization (Antelo et al., 2007). It must be remarked some extra control loops are needed to achieve the convergence of the intensive variables, since the inventory control by itself does not ensure the convergence of these variables to a desired operation point. This TBC approach allows the designer to develop a superstructure containing all the stable control alternatives. The best control structure in this set as well as the controller's tuning parameters will be optimally selected in order to minimize the total cost of the system while satisfying a large number of feasibility constraints in the presence of time-varying disturbances. The arising optimization problem is a challenging mixed-integer dynamic optimization (MIDO) problem. In this work, we present a new method called MITS, which is based on extensions of the metaheuristic Tabu Search (TS) for solving these problems.

Finally, the proposed TBC design approach will be illustrated on the challenging benchmark of the Tennessee Eastman Process (Downs and Vogel, 1993).

The paper is structures as follows: In Section 2, the fundamentals of the TBC design approach are presented, including the process network formalism as well as the thermodynamic basis of the conceptual inventory control. In Section 3, we present the realization of this conceptual part into real control loops, establishing a MINLP problem to optimal tune and to select the best control structure candidate between a given set of alternatives, which is solved by using a new optimization algorithm (MITS).

# 2. The Thermodynamic-Based Control Design applied to the TEP

We apply here the thermodynamic-based control (TBC) design to derive robust decentralized controllers for the Tennessee Eastman Process, ensuring the global stability of the plant. In this framework, the TEP is represented as a process network (PN) composed of coupled mass and energy inventory layers as depicted in Figure 1. In this Figure, each phase present in the process is represented by one circle denoting a node, and solid circles symbolize the environment (node 0). As stated in previous work by the authors, a node is a well mixed homogeneous material region. To each node j in the network, we associate a state vector  $z_i \in \mathbb{R}^{+(c+1)}$  of the form  $z_i = (n_i^j, ..., n_c^j, u^i)^T$ , where  $n_i^j$  denotes the mole numbers of component i, the internal energy is represented by  $u^{j}$  and c stands for the total number of chemical species (for the TEP case,  $c = \delta$ ). Nodes and environment are connected by a set of n convective fluxes we refer to as  $f_i \in \mathbb{R}^{+c}$  and  $p_i(f_i) \in \mathbb{R}^+$  for component and energy, respectively. These mass and energy convective flows are represented by solid and dashed arrows, respectively. Nodes in the network can also be interconnected by dissipative transfer fluxes (due to thermodynamic potentials) collected in vectors  $\varphi^k \in \mathbb{R}^{+d_c}$  (with k = 1,...,c) and  $\psi$  $\in \mathbb{R}^{+d_u}$ , where  $d_c$  and  $d_u$  stand for mass and energy dissipative transfer, respectively. In Figure 1, these dissipative fluxes are represented by solid and dashed double-head arrows for mass and energy, respectively, while convective flows are denoted by solid (mass) and dashed (energy) single arrows. Finally, the inventory network dynamics for a dissipative subnetwork  $\mathcal{D}$  (which is constituted by the collection of  $\theta_i$  nodes only interconnected through dissipative fluxes) can be represented by:  $\dot{m}_I = \mathcal{N}\phi_I$ ;  $\dot{u}_I =$  $\mathcal{N}p_l$ , with  $\dot{m}_l, \dot{u}_l, \phi_l, p_l \in \mathbb{R}^{+\ell}$ , where  $m_l = \sum_{i \in \mathcal{D}} \sum_{k=1}^c \sigma^k n_i^k$  is the total mass

inventory,  $\sigma^k$  represents the vector of molecular weights for the c components.  $\phi_I = \sum_{i \in \mathcal{D}} \sum_{k=1}^c \sigma^k f_i^k$  and  $p_I = \sum_{i \in \mathcal{D}} p_i$  are the mass and energy inventory flows, respectively.  $\mathcal{N} \in \mathbb{R}^{\ell \times \ell}$  is the convective matrix and shows the connections between nodes and with the environment, respectively, being  $\ell$  the number of dissipative subnetworks (for the TEP network in Figure 1,  $\ell = 6$ ).

# **2.1. The thermodynamic fundamentals:** In every node of a process network with volume $v_i$ , thermodynamics provides us with a continuous, twice differentiable scalar

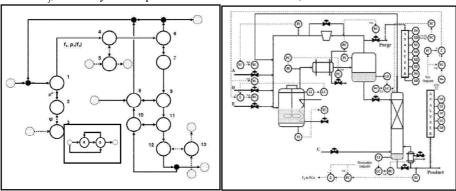


Figure 1. Process Network for the TEP

Figure 2. TEP flowsheet and TBC structure by Antelo et al. (2006)

function  $S_j(z_j, v_j)$ :  $\mathbb{R}^{+(c+2)} \mapsto \mathbb{R}$  which corresponds with *entropy* (Alonso and Ydstie, 1996). This function is first order homogeneous in all its arguments and strictly concave with respect to the vector  $z_j$ . Such property indicates that S has a definite curvature. The main control objective is to ensure the convergence of the network inventories in  $\mathcal{D}$  to a compact convex set  $\Lambda$  defined by constant inventories, because it is over this set where the entropy reaches a maximum for given constant total mass, energy and volume  $(S_{max})$ . Once the system reaches the interior of this region  $\Lambda$  (not the bounds where  $z_j = 0$ ), we can ensure that the process system is passive and stability can be proved (Ydstie and Alonso, 1996; Alonso and Ydstie 1997) making use of a formal framework for stability analysis of open process systems, in which the difference  $S_{max} - S$  can be used as storage function candidate. This fact motivates a hierarchical control design decomposition in which *conceptual mass and energy inventory control loops* in every node of the network are first designed to drive the system to this compact set  $\Lambda$ . In particular, the mass and energy inventory control layers consist of linear proportional controllers of the form:  $\phi_I = \phi_I^* + \omega_m(m_I - m_I^*)$ ;  $p_I = p_I^* + \omega_u(u_I - u_I^*)$ .

In order to exemplify how to define this *conceptual inventory control loops*, let us now consider that the process network corresponding to the TEP (Figure 2) can be simplified into a new network constituted by two nodes representing the reaction (*R-node*) and the separation (*S-node*) parts of the process, as presented in the box on Figure 2 for the vapor mass layer of the TEP network. As a general rule, we will use whenever possible the total inventory flow leaving each node of the network. Therefore, the reactor and separator vapor mass loops will be closed by acting over the *R-node* and *S-node* outflows, respectively. The same logic applies to the liquid mass and energy layers.

We show next the way this conceptual design is realized, establishing the way to select the best control candidate between a given subset of alternatives which belongs to the global superstructure of the TEP as well as tuning the *PI* controller parameters.

## 3. The conceptual inventory control realization

The next step in the TBC design is to *realize* the proposed conceptual inventory control loops using the physical inputs-outputs of the process since the total inventory fluxes can be the result of combining multiple convective outflow streams. This fact leads to several alternatives regarding the control structure design of the TEP, which will be form a *superstructure* of control candidates. By considering this, the mathematical form of the problem can be stated as follows:

$$\min_{v} J(v, t_f)$$
s.t.  $f(\dot{x}, x, p, v) = 0; x(t_0) = x_0; h(x, p, v) = 0; g(x, p, v) \ge 0; v_l \le v \le v_u$  (1)

where v is the vector of decision variables, J is the objective function (often representing the costs) to minimize, x is the state vector, and f is the set of differential and algebraic equality constraints describing the system dynamics. Finally, h and g are possible equality and inequality path and/or point constraints which express additional requirements for the process performance. The lower and upper bounds for the decision variables are given with  $v_l$  and  $v_u$ . The common way to express the alternatives which form the superstructure is by introducing binary variables. As a result the problem formulated in (1) is a mixed-integer dynamic optimization (MIDO) problem.

For the TEP control design,, all the TBC structures inside this superstructure satisfy that the inventories in the process will converge to a given convex set, and therefore output stability is guarantee. In order to analyze the control alternatives for the highly nonlinear reactor in the TEP, we have considered a subset of the global set of candidates, which are derived from the original TBC control design by Antelo *et al.* (2006) (Figure 2): i) Concerning the reactor level control loop, its set point modifies the reference for the flow controller acting over the *E* feed. As an alternative, *D* Feed is proposed; ii) For the reactor pressure case, the original proposal by Antelo *et al.* (2006) considers the condenser cooling water flow as the manipulated variable plus a control over the vapor mass inventory in the separator by using the purge rate, ensuring that all inventories in the TEP will remain bounded. As an alternative to close this level loop, we propose the purge flow. When this case is considered, an extra loop over the separator temperature (energy inventory) by using the condenser coolant flow is defined.

In order to determine the best control alternative among the proposed ones, a new binary vector b is added to our system dynamics. These 0-1 variables express which of the four control strategies is being used, and they are defined as:  $b_1 \in \{0, 1\}$  (E feed);  $b_2 \in \{0, 1\}$  (D feed);  $b_3 \in \{0, 1\}$  (Condenser coolant) and  $b_4 \in \{0, 1\}$  (Purge flow). Therefore, the TBC design proposal by Antelo *et al.* (2006) will be characterized by the vector  $b = (I, 0, I, 0)^T$ . According to this discussion, the optimization problem employed to set the best alternative translate into the following MINLP:

$$\min_{v,b} J(x,v,b)$$
s.t.  $f(\dot{x},x,p,v,b,t) = 0$ ;  $h(x,p,v,b) = 0$ ;  $g(x,p,v,b) \ge 0$ ;  $b_1 + b_2 = 1$ ;
$$b_3 + b_4 - 1 \ge 0$$
;  $v_l \le v \le v_u$ ;  $b_l \le b \le b_u$  (2)
where  $b \in \{0,1\}^4$  is the vector of binary variables (0-1 variables) and  $v \in \mathbb{R}^{36}$  are the continuous variables (the *PI controller parameters*). The lower and upper bounds for the binary variables will be of the form  $b_l = (0, ..., 0)^T$  and  $b_u = (1, ..., 1)^T$ . It must be

pointed out that we are considering that only one of the two alternatives for each loop

can be active at one time, being necessary to introduce additional linear constraints related to vector b. The MINLP is also made up of the following constraints which are related with the reactor pressure, temperature and volume, and with the separator and the stripper volumes ( $P_{reactor} \leq 3000 \, KPa$ ;  $2 \, m^3 \leq V_{reactor} \leq 24 \, m^3$ ;  $T_{reactor} \leq 175 \, ^{\circ}\text{C}$ ;  $1 \, m^3 \leq V_{separator} \leq 12 \, m^3$ ;  $1 \, m^3 \leq V_{stripper} \leq 6 \, m^3$ )

The objective function considered is the one proposed by Downs and Vogel (1993) in the TEP definitions. It is based on the operating costs and can be defined as follows:

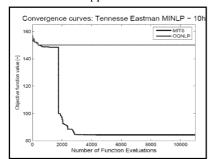
$$TC = \underbrace{7.5973 \frac{\$}{Kmol}}_{PC} \cdot PR + \underbrace{0.1434 \frac{\$}{Kmol}}_{PC} \cdot PrR + \underbrace{0.0536 \frac{\$}{kW \cdot h}}_{CC} \cdot CW + \underbrace{0.0318 \frac{\$}{Kg}}_{SC} \cdot SR$$
(3)

where TC are the total operating costs at the base case, PC and PR are the purge costs and purge flowrate, respectively. Analogously, PrC, CC and SC are the costs associated to the product stream, compressor and steam, and PrR, CW and SR are the product rate, the compressor work and the steam rate, respectively. Note that the objective function used in the MINLP formulation will be the mean of these operating costs along the whole simulation time horizon. For this work, this simulation time horizon was set to t = 10 h. This is enough time for stabilization of the TEP.

After all these considerations concerning the objective function, the problem can be represented as an MINLP of the form (1):

$$v \in \mathbb{R}^{36}$$
,  $b \in \{0,1\}^4$ : min  $\overline{J(x,v,b)} = \overline{TC}$  with  $v_0 - 0.5v_0 \le v \le v_0 + 0.5v_0$  (4)

The lower and upper bound for the decision variables (v) have been set to be the  $\pm 50\%$ 



of the initial value for the decision vector to avoid as much as possible the valve saturation problems. Note that changes in the decision variables will be translated into variations in the states x that can even drive the system to shutdown due to the fact that one or more of the constraints have been violated. In order to solve the MINLP (Eq. 4) problem, many algorithms for global optimization use a local solver to identify a local minimum by starting from an initial point, and in order to reach the

Figure 3.TEP Convergence curves from an initial point, and in order to reach the global minimum, a special strategy for deciding where to start the local solver is used. As the global component, a procedure based on extensions of the Tabu Search (TS) algorithm is applied. Regarding the local solver, we have used an adaptation of TS to mixed-integer nonlinear optimization problems, called *Mixed-Integer Tabu Search* (MITS). Regarding the local solver, we have used a special adaptation of a sequential quadratic programming method for the mixed-integer case called MISQP (Exler and Schittkowski, 2006). We compare this new solver with a well-established one as OQNLP. The dynamic model for the proposed TBC control design was implemented as a SIMULINK code. Table 1 lists the default values (initial point) and the best point located by MITS. The solution obtained by MITS for the binary vector is  $b = (0, 1, 0, 1)^T$ . This vector defines the new realization of the control loops for the pressure and the level in the reactor by acting over the purge and the D Feed, respectively. Figure 3

shows the convergence curves for both solvers. MITS clearly outperformed OQNLP, both regarding the final objective function value and the convergence speed.

Table 1. Optimal tuned parameters and best control structure obtained by solving Eq. (4)

Decision Variable	Control Loop	Default Values	Result MITS	v(21)	D Feed Flow	$1.6667e^{-5}$	$1.4535e^{-5}$
v(1)	A Feed Flow	0.001	$1.4029e^{-3}$	v(22)	E Feed Flow	4.1667	3.1266
v(2)	D Feed Flow	0.003	0.004043	v(23)	C Feed Flow	0.1667	0.83334
υ(3)	E Feed Flow	$1.8e^{-10}$	$9.6882e^{-31}$	v(24)	Condenser Coolant	4.1667	2.0834
v(4)	C Feed Flow	20	30.0	v(25)	Separator Flow	$1.6667e^{-5}$	$1.42116e^{-5}$
v(5)	Condenser Coolant	7e-7	$9.8858e^{-7}$	v(26)	Stripper Flow	$1.6667e^{-5}$	$1.302e^{-5}$
v(6)	Separator Flow	$4e^{-4}$	$4.5727e^{-4}$	v(27)	Production Rate	2	1.2995
υ(7)	Stripper Flow	0.004	0.004242	v(28)	Stripper Level	0.3333	0.19213
v(8)	Production Rate	3.2	4.8	v(29)	Separator Level	3.3333	1.6704
v(9)	Stripper Level	-0.02	-0.021864	v(30)	Reactor Level	0.01667	0.011874
v(10)	Separator Level	-0.05	-0.048376	v(31)	Reactor Pressure	0.3333	0.18142
v(11)	Reactor Level	10	8.5024	v(32)	%G in Product	1.6667	1.0927
v(12)	Reactor Pressure	-0.0001	-5e <sup>-5</sup>	v(33)	%A in Parge	9.3667	10.284
v(13)	%G in Product	-0.032	-0.02811	v(34)	Recycle Rate	25	30.338
v(14)	%A in Purge	0.0009	$5.664^{-4}$	v(35)	Reactor Temp.	0.125	0.06475
v(15)	Recycle Rate	0.00125	0.001271	v(36)	Separator Temp	8.3333	6.4701
v(16)	Reactor Temp.	-8	-8.1636	b(1)	Reactor Level (E Feed)	1	0
v(17)	Separator Temp.	100	90.219	b(2)	Reactor Level (D Feed)	0	1
v(18)	G/H Product Ratio	32	21.762	b(3)	Reactor Press. (Cond. Coolant)	1	0
v(19)	G/H Product Ratio	46	30.891	b(4)	Reactor Press. (Purge Rate)	0	1
v(20)	A Feed Flow	1,6667e 5	1.584e <sup>8</sup>	Cost Value (\$/h)		156.843	84.289

#### 4. Conclusions

In this contribution, a new systematic plant-wide control design methodology called TBC has been applied to the challenging benchmark of the Tennessee Eastman Process. All the hierarchical designs derived from the TBC approach guarantee the global stability of the TEP. Finally, a MINLP optimization problem was defined to optimal tune the PI controller parameters as well as to select the best candidate inside the set of TBC candidates. We have introduced a new algorithm (MITS) to solve this MINLP, which has presented very good performance.

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