# Heat Exchanger Network Synthesis-A Bilevel Decomposition Method Based on Stream Data Grouping

Timo Laukkanen and Carl-Johan Fogelholm

Helsinki University of Technology, Energy Engineering and Environmental Protection, P.O. Box 4400, FIN-02015 HUT, Finland

Different tools and methods have been presented to solve the Heat Exchanger Network Synthesis (HENS) problem. In this work a simultaneous HENS method that uses bilevel optimization, stream data grouping and aggregation of streams is presented. The results obtained using the method compare well with the results obtained with another simultaneous HENS method. For bigger problems, especially if an approximate solution is acceptable, this can be accomplished also with less computational work.

## 1. Introduction

Heat exchanger network synthesis (HENS) has been an active research area for more than 40 years. This is mainly due to its importance in cost-efficiently achieving energy savings in industrial processes. The problem is also hard to solve. As Furman and Sahinidis (2001) prove, the problem is NP-hard and hence there probably exists no computationally efficient (polynomial) algorithm to solve the problem. Thus methods that provide good approximate solutions are useful. A lot of different tools and methods have been presented to solve the HENS problem. An extensive review of these methods can be found in Furman and Sahinidis (2002).

Probably the best known and still one of the best HENS methods is the Pinch analysis (f.ex. Linnhoff & Hindmarsh, 1983). In pinch analysis different targets (minimum utility consumption, minimum number of units and minimum heat transfer area) of HENS are solved sequentially. Mathematical programming has been used with this sequential strategy (f.ex. Papoulias and Grossmann, 1983). Pettersson (2005) developed a computationally efficient sequential strategy, where the terms involved in the objective function are more accurate in each sequential step. Because the sequential nature of the methods can cut off the best solutions, simultaneous methods formulated as a mixed integer non-linear programming (MINLP) model, have been developed (f.ex. Yee and Grossmann, 1990). Although the simultaneous methods are rigorous they typically are computationally time-consuming. Bilevel optimization was used by Iyer and Grossmann (1998) in synthesis and operational planning of utility systems. They showed that bilevel optimization can be useful in process synhesis problems especially regarding the computation times.

In this work a simultaneous HENS method that uses bilevel optimization, stream data grouping and aggregation of streams is presented. The objective of the method is to generate good, and for small problems, rigorous solutions for the HENS problem computationally as efficiently as possible.

## 2. Method

Figure 1 depicts the overall method. The problem has a set of hot and cold streams that have to be heated or cooled from fixed starting temperatures to fixed target temperatures. Given are also heat capacity flowrates for these streams. Hot and cold utilities are available with fixed start and target temperatures. Cost data for the utilities and the investment costs associated for installing the heat exchangers is given.

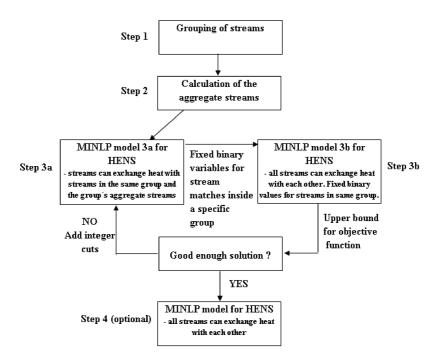


Figure 1: The overall method

## 2.1 Step 1, grouping of streams

The first step is to decide into how many groups the streams are grouped to. In the proposed method the minimum number of groups is 2 and the maximum is equal to the smaller value of the number of hot or cold streams. It is better to have at most 10 streams in a group and as few groups as possible.

The actual grouping of streams is done so, that for every stream the investment cost of installing a heat exchanger, if it exchanged heat with a reference stream, is calculated. The investment costs include the unit installation costs and the cost associated with heat transfer area. The reference stream used is the cold utility for hot streams and the hot utility for the cold streams. According to these investment costs, the hot streams are

sorted in descending order, and divided into the groups with approximately equally many streams in all groups. The cold streams are sorted in ascending order with respect to their investment costs, and as with the hot streams, divided into groups with approximately equally many streams in all groups. Hence, the hot streams with high investment costs and cold streams with low investment costs are in the same group.

#### 2.2 Step 2, calculation of the aggregate streams

Cold and hot aggregate streams are made from all the cold and hot streams outside a specific group. For example if there are 3 groups altogether, 3 cold and 3 hot aggregate streams are made. If there are also 3 hot streams altogether, one in each group and stream 1 is the only hot stream in group 1, the hot aggregate stream of group 1 represents hot streams 2 and 3. The aggregate stream should have approximately the same area if it exchanged heat with a reference stream (cold utility for hot aggregate streams and hot utility for cold aggregate streams) as the sum of areas of all the streams (the ones that the aggregate stream represents) had if they exchanged heat with a reference stream. The heat of the aggregate stream is equal to the sum of heats of the streams that the aggregate stream is representing. Currently in the method the starting and target temperatures of a cold aggregate stream is fixed to equal the lowest starting temperature and highest target temperature of the cold streams that the cold aggregate stream represents. The starting and target temperatures of a hot aggregate stream are fixed to equal the highest starting temperature and the lowest target temperature of hot streams that the hot aggregate stream represents. Hence, also the heat capacity flowrates are also fixed. The only positive variable that can be optimized is the heat transfer coefficient of every aggregate stream. A nonlinear programming (NLP) model is used to calculate the aggregate streams.

## 2.3 Step 3a, MINLP model 3a for HENS

In step 3a, a HENS model for each group is solved. In the model hot streams can exchange heat with cold streams that are in the same group, with the group's cold aggregate stream and with the cold utility. Cold streams can exchange heat with hot streams that are in the same group, with the group's hot aggregate stream and with the hot utility. Aggregate streams can exchange heat with the streams in the same group, and with utilities, but no utility costs nor investment costs are associated with aggregate stream and utility heat exchanging. After a solution has been obtained the binary variables that define an existence of a match between hot and cold streams in the same group are fixed.

## 2.4 Step 3b, MINLP model 3b for HENS

In step 3b, a HENS model of the overall problem is solved with the fixed binary variables for the matches of hot and cold streams in the same group. All other variables are free to vary unless bounds have been set for them. The result of Step 3b is an optimized but approximate solution. Currently in the method, a solution is regarded good enough if the solution of model in step 3a has increased compared to a previous result of model in step 3a.

## 2.5 Step 4, MINLP model 4 for HENS (optional)

If the problem in hand is not too big, model 4 of step 4 can be used to check and possibly improve the solution of step 3b. The HENS model of step 4 is a rigorous model, where the only restriction (besides possible constraints given by the user of the model) is that the solution has to be smaller than the solution of step 3b. This should decrease the solution time.

#### 3. Results

The presented method has been used to solve two small HENS examples. The problems have bee solved with GAMS (Brooke et al., 1992). The SYNHEAT- superstucture and model by Yee and Grossmann (1990) is used in all 4 steps of the method, although in steps 1 and 2, the model was simplified slightly. DICOPT++ (Viswanathan and Grosmann, 1990) is the local MINLP-solver used. CONOPT3, from ARKI Consulting and Development A/S, is the NLP-solver used in DICOPT++ and in purely nonlinear models. CPLEX® is the MILP-solver. BARON (Sahinidis, 2000) with CPLEX® and MINOS 5.0 (Murtaugh and Saunders, 1983) is used when globally optimal solutions are required. The computer used is a mobile Intel® P III 1 GHz.

Example 1 has been taken from Zamora and Grossmann (1998). The upper part of Table 1 gives the stream data for this example. Heat exchanger cost function for process heat exchangers and coolers is  $15000 + 30a^{0.8}$  (\$ per year, a = area in m<sup>2</sup>) and for heaters  $15000 + 60a^{0.8}$  (\$ per year, a = area in m<sup>2</sup>). Hot utility costs 110 and cold utility 10 (\$ kW<sup>-1</sup> per year). The problem is solved with the presented method with DICOPT (2 groups). The optional step 4 is then solved with BARON. Hence the global optimum can be guaranteed. The objective function value is 422.7 k\$ after step 3b, and 415.2 k\$ after step 4. The calculation times are 3.0 s after step 3 and 473.4 s after step 4. In order to compare the results of the presented method, the problem was also solved with the basic SYNHEAT-model with DICOPT and with BARON, respectively. Then the objective function value was 426.3 k\$, when DICOPT was used and 415.2 k\$ when BARON was used. The calculation times were 1.0 s with DICOPT and 540.2 s with BARON. The EMAT (Minimum approach temperature), has a given lower bound of 1. All models have 2 stages in the superstructure. When comparing these results to the ones presented in Björk and Westerlund (2002), global results (Step 4) are global indeed (for the used superstructure) and the results with a local solver (Step 3) are comparative as well.

Example 2 has been presented by Linnhoff and Ahmad (1990). The lower part of Table 1 gives the stream data for this example. Heat exchanger cost function for all heat exchangers is  $2000 + 70a^{1.0}$  (\$ per year, a = area in m²). Hot utility costs 60 and cold utility 6 (\$ kW¹¹ per year). The problem is solved with the presented method in Case 1 with DICOPT (3 groups). The optional step 4 is solved with DICOPT. The objective function value is 2955.3 k\$ after step 3 and after step 4 the result did not improve. The calculation times are 24.8 s after step 3 and 30.5 s after step 4. In order to compare the results of the presented method, the problem was also solved with the basic SYNHEAT-model with DICOPT. Then the objective function value is 2989.0 k\$ and the solution

time is 119.3 s. EMAT has a lower bound of 1.0 and hot utility usage has an upper bound of 27000 kW. All models have 4 stages in the superstructure. When comparing these results to the ones presented in Pettersson (2005), the results are comparative, although for this specific problem, the sequential methods are at least as good. This can stem from the superstructures used in the SYNHEAT method and in the presented method.

Table 1:Stream data for examples 1 and 2.

	Stream	Tin ( <sup>0</sup> C)	Tout ( <sup>0</sup> C)	FCp (kW/K)	h (kW/m <sup>2</sup> K)
Example 1	H1	180	75	30	0.15
	H2	240	60	40	0.10
	C1	40	230	35	0.20
	C2	120	300	20	0.10
	Hot Utility	325	325	-	2.00
	Cold Utility	25	40	-	0.50
Example 2	H1	327	40	100	0.50
	H2	220	160	160	0.40
	H3	220	60	60	0.14
	H4	160	45	400	0.30
	C1	100	300	100	0.35
	C2	35	164	70	0.70
	C3	85	138	350	0.00
	C4	60	170	60	0.14
	C5	140	300	200	0.60
	Hot Utility	330	250	-	0.50
	Cold Utility	15	30	-	0.50

## 4. Conclusions

A new method for HENS has been presented. The method uses bilevel decomposition, stream data grouping and aggregation of streams in a systematic manner. The results obtained using the method compare well with the results obtained with other methods and especially with the basic SYNHEAT-model. The SYNHEAT-model uses the same superstructure as the steps in the presented method, and hence has the same assumptions, limitations and strengths. This way the basic SYNHEAT-model is the most appropriate baseline for comparison for the presented method. As can be seen from the results, at least as good results (total annual cost) can be obtained with the presented method, and especially for bigger problems and if approximate results are appropriate, with much less computational effort.

The method could be improved in the future. The grouping and building of the aggragate streams could be combined into one step. It would be beneficial if the solution of step 3a (the HENS model of separate groups) could be forced to be a lower bound for the total problem. Then the gap between the lower bound (Step 3a) and upper bound (Step 3b) could be seen. Other superstuctures that do not use the stage-wise superstructure could be used as the base superstructure in the different steps. Then still more accurate solutions could be otained. In the future the method will be used in an interactive multiobjective optimization tool for HENS. Due to the interactive nature, the solution times can not be too long and hence the presented method should be very

approprriate. It would also be interesting to apply the idea of data grouping, aggregate models and bilevel optimization to other process synthesis problems.

## 5. References

- Björk, K.-M., and Westerlund, T. 2002, Global optimization of heat exchanger network synthesis problems with ans without the isothermal mixing assumption. Computers and Chemical Engineering. 26, 1581-1593.
- Brooke, A., Kendrick, D., and Meeraus, A., 1992, Gams: a user's guide. The Scientific Press
- Furman, K., and Sahinidis, N., 2001, Computational complexity of heat exchanger network synthesis. Computers and Chemical Engineering. 25, 1371-1390.
- Furman, K., and Sahinidis, N., 2002, A critical review and annotated bibliography for heat exchanger network synthesis in the 20th century. Industrial Engineering Chemical Research, 41, 2335-2370.
- Iyer, R R., and Grossmann, I. E., 1998, Synthesis and operational planning of utility systems for multiperiod operation. Computers and Chemical Engineering. 22(7-8), 979-993.
- Linnhoff, B., and Ahmad, S., 1990, Cost optimum heat exchanger networks- 1. Minimum energy and capital using simple models for capital cost. Computers and Chemical Engineering, 14(7), 729-750.
- Linnhoff, B., and Hindmarsh, E., 1983, The pinch design method for heat exchanger networks. Chemical Engineering Science, 38, 745-763.
- Murtaugh, B. A., and Saunders, M. A., 1983, MINOS 5.0 user's quide. California: Systems Optimization Laboratory, Department of Operations Research, Standford University.
- Papoulias, S. A., and Grossmann, I. E., 1983, A structural optimization approach in process synthesis- II, heat recovery networks. Computers and Chemical Engineering. 7(6), 707-721.
- Pettersson, F., 2005, Synthesis of large-scale heat exchanger networks using a sequential match reduction approach. Computers and Chemical Engineering. 29, 993-1007.
- Sahinidis, N. V., 2000, Baron, branch and reduce optimization navigator, user's manual version 4.0.
- Viswanathan, J., and Grossmann, I. E., 1990, A combined penalty function and outer approximation method for MINLP optimization. Computers and Chemical Engineering. 14, 769-782.
- Yee, T. F., and Grossmann, I. E., 1990, Simultaneous optimization models for heat integration- II. Heat exchanger network synthesis. Computers and Chemical Engineering. 14(10), 1151-1164.
- Zamora, J. M., and Grossmann, I. E., 1998, A global MINLP optimization algorithm for the synthesis of heat exchanger networks with no stream splits. Computers and Chemical Engineering. 22, 367-384.