

Interval Based Minlp Superstructure Synthesis Of Heat Exchange Networks

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An interval based mixed integer nonlinear program (MINLP) superstructure model for the synthesis of heat exchange networks (HEN) is presented. The intervals of the superstructure are defined by the supply and target temperatures of either the hot or cold set of streams (including utilities). Heat can be exchanged between each hot and cold stream within each interval. This model can simultaneously trade-off energy, heat transfer area and number of units while at the same time generating a close to optimal network structure. Constraints on matches can easily be handled as well as streams with significantly different heat transfer coefficients. This model unlike other simultaneous HENS models automatically mixes streams at equal temperatures even without the inclusion of mixing equations, thus reducing the solution time. Intermediate utilities can also easily be included in the superstructure. The model is applied to two example problems in the literature and it is shown to produce satisfactory results. A special feature of this method is that no special initialisation procedure is required in order to obtain the optimal solution.

1. Introduction

The synthesis of heat exchanger networks has been accomplished using methods based on physical insights and mathematical programming (MP). Pinch technology (PT) has been the most dominant under the physical insights approach. The minimum temperature difference (ΔT_{\min}) is the key variable for establishing the trade-off among the competing costs (see Shenoy, 1995). The PT method due to its sequential nature does not establish a simultaneous trade-off among the competing costs, each step of the optimisation is dependent on previous ones, this becomes more tedious for situations in which intermediate utilities are involved. Shenoy et al. (1998) established targets for multiple utility selection through the use of the cheapest utility principle (CUP) which favours the use of the cheapest utility as the total network energy usage increases. The approach involves fixing the ΔT_{\min} and then varying the use of each utility so as to determine which utility (or set of utilities) give the least operating and associated capital costs. In as much as this might produce a good total annual cost (TAC) target, meeting such targets in design would be time consuming especially for large industrial problems because PT does not have a fixed procedure as far as the application of the pinch design tools such as the driving force plot and the remaining problem analysis are concerned.

The MP approach does not involve segregating the synthesis process into steps as is the case with the pinch method. Instead a superstructure (or hyperstructure) which embeds all

possible networks is set up and subsequently optimised in order to determine the best network for a heat exchange problem. With this approach the costs contributing to the TAC can be simultaneously traded off. The MP approach though simultaneous in its synthesis does not have a guarantee of optimality in its solution due to the non convexities of HENS problems (Shenoy, 1995).

2. Superstructure model

The interval-based MINLP superstructure (IBMS) is illustrated by Figure 1 for a problem with two hot and two cold streams.

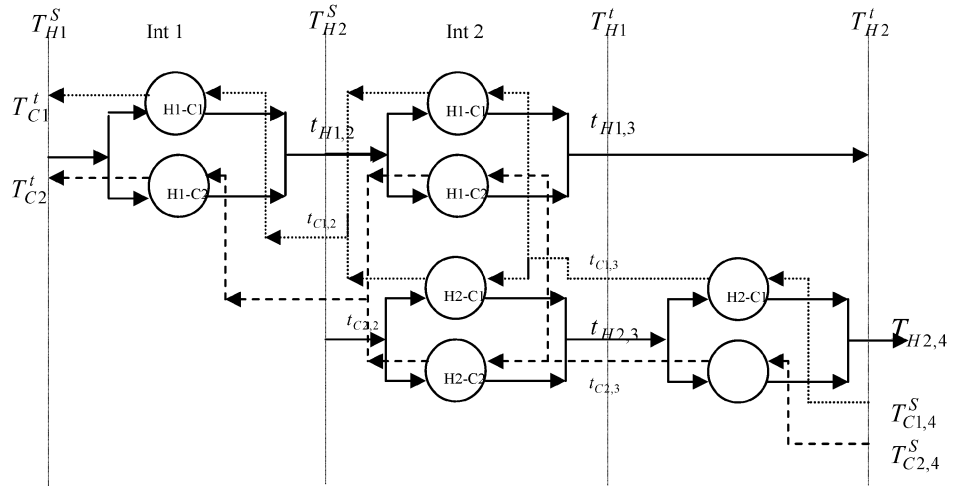


Figure 1. Interval-based heat exchanger network superstructure

In Figure 1, the superstructure intervals are defined by the supply T_H^S and target T_H^I temperatures of the hot streams (process and utilities) only. These temperatures decrease from left to right. The cold streams (process and utilities) are assumed to participate in all intervals, however constraints are used to ensure thermodynamic feasibility for heat exchange. Note that this order could be reversed such that the cold streams supply T_c^S and target T_c^I temperatures are used to define the intervals while the hot streams are assumed to participate in all the intervals. Similar to the stagewise superstructure of Yee and Grossmann (1990), each stream in each interval is split and has the potential to exchange heat with each stream of the opposite kind only once. Also, the temperatures of each hot and each cold stream are treated as variables at all intervals other than those that correspond to their supply/target temperatures.

The IBMS is different from the stagewise superstructure of Yee and Grossmann (1990) where none of the hot or cold streams supply/target temperatures are used to define stage

temperatures other than the first and last boundary stages in the stagewise superstructure. The total number of stages of the stagewise superstructure is chosen to correspond to the maximum of the hot or cold streams, however more stages can still be used. Also, the stagewise approach of Yee and Grossmann (1990) needs to be solved first for an MINLP for situations in which the superstructure contains split streams, such configurations are further solved in an NLP suboptimisation step in order to determine the optimal split flow ratios. Isothermal mixing is not always obtained in the first MINLP optimisation step of the stagewise superstructure because stage temperatures other than the first and last of the superstructure are all modelled as variables.

Setting the intervals of the superstructure to correspond to the supply and target temperatures of either the hot or cold streams helps to eliminate the need for isothermal mixing equations in that split streams are automatically mixed at equal temperatures which correspond to the interval temperature of the stream concerned. This is demonstrated in the example problems. Overall heat balance equations are set for each hot and cold stream as well as interval heat balance. These are shown in Equations 1 and 2 below,

$$(T_i^s - T_i^t)F_i = \sum_{m \in M} \sum_{j \in C} q_{ijm} \quad i \in H \quad (1)$$

$$(T_j^t - T_j^s)F_j = \sum_{m \in M} \sum_{i \in H} q_{ijm} \quad j \in C \quad (2)$$

T_i^s and T_i^t are the supply and target temperatures of hot stream i belonging to the set H while T_j^s and T_j^t are the supply and target temperatures of cold stream j belonging to the set C . Index m represents an interval belonging to the set M , q_{ijm} and F which are continuous variables are the heat exchanged between hot stream i and cold stream j in interval m and the heat capacity flowrate of each stream respectively. Equations 3 and 4 below describe the heat balance for each hot and each cold stream in intervals in which they participate,

$$(t_{i,m} - t_{i,m+1})F_i = \sum_{j \in C} q_{ijm} \quad m \in M \quad (3)$$

$$(t_{j,m} - t_{j,m+1})F_j = \sum_{i \in H} q_{ijm} \quad m \in M \quad (4)$$

where $t_{i,m}$ and $t_{j,m}$ are continuous variables which correspond to the temperatures of hot stream i and cold stream j in intervals other than their supply and target temperatures. Constraints are further used to define the first and last boundary temperatures of the superstructure. Note that in Figure 1, this corresponds to $T_{H1,1}^s$ for the first boundary and $T_{C1,4}^s$ and $T_{C2,4}^s$ for the last. Temperatures need to decrease from left to right for both hot and cold streams, these are defined with constraints, however each hot stream (from Figure 1) is constrained to only decrease starting from the interval which corresponds to its supply temperature. Binary variables, y_{ijm} are used to represent the existence of a match (i,j) in interval m . An upper bound, Ω , is used alongside y_{ijm} to constrain the amount of heat

exchanged for matches that exist in interval m . Ω is set as the smaller of the heat loads of the two streams involved in the match. Equation 5 below describes this,

$$q_{ijm} - \Omega y_{ijm} \leq 0 \tag{5}$$

Approach temperatures dt (which are also continuous variables) are used to model the driving forces of the hot and cold ends of each interval in order to calculate the heat exchange areas. These variables are included within the objective function so as to reduce the nonlinearities in the model. The binary variables, y_{ijm} , are used alongside dt to activate the driving forces for the matches that exist, while Φ (which can be calculated from stream data) is included as an upper bound for the driving forces. Equations 6 and 7 illustrate this,

$$dt_{ijm} \leq t_{i,m} - t_{j,m} + \Phi(1 - y_{ijm}) \quad m \in M, \quad i \in H, \quad j \in C \tag{6}$$

$$dt_{ijm+1} \leq t_{i,m+1} - t_{j,m+1} + \Phi(1 - y_{ijm}) \quad m \in M, \quad i \in H, \quad j \in C \tag{7}$$

Note that the Paterson (1984) approximation is used to calculate the logarithmic mean temperature difference (*LMTD*). dt_{ijm} is set close to a small positive value, the exchanger minimum approach temperature (*EMAT*). The objective function simultaneously minimises the utility costs, the number of units and the heat exchange areas. Equation 8 presents this,

$$\begin{aligned} \min \quad & \sum_{i \in P} \sum_{m \in M} CU q_{ijm} + \sum_{j \in C} \sum_{m \in M} HU q_{ijm} + \sum_{i \in H} \sum_{j \in C} \sum_{m \in M} CB_{ij} y_{ijm} \\ & + \sum_{i \in H} \sum_{j \in C} \sum_{m \in M} A [q_{ijm} / U_{ij} \{LMTD\}]^{D_{ij}} \end{aligned} \tag{8}$$

U is the overall heat transfer coefficient, CU and HU are the costs of the hot and cold utilities, CB is the fixed charge for exchangers, A is the area cost coefficient and D is the area index.

Example problem 1

This example is from Yee and Grossmann (1990). It involves six hot streams and two cold streams (see Table 1). It will be used to demonstrate the mixing of streams at equal temperatures even when no mixing equations are included in the superstructure model. Figure 2 presents the optimal network for the IBMS.

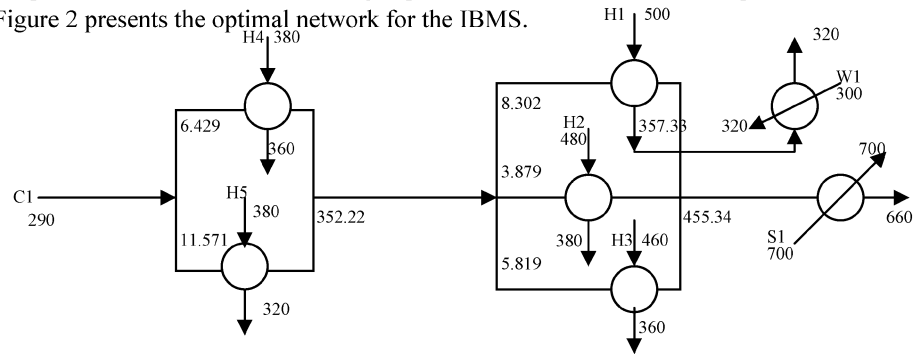


Figure 2. IBMS network for Example 1 showing isothermal mixing.

Table 1: Stream and capital cost data for Example 1

Stream	T^s (K)	T^t (K)	F (kW K ⁻¹)	Costs
H1	500	320	6	-
H2	480	380	4	-
H3	460	360	6	-
H4	380	360	20	-
H5	380	320	12	-
C1	290	660	18	-
S1	700	700	-	140
W1	300	320	-	10

$U=1$ (kW m⁻² K⁻¹) for all matches, Annualised cost = $1200[\text{Area}(\text{m}^2)]^{0.6}$ for all exchangers, utility costs in \$ kW⁻¹ yr⁻¹.

The configuration in Figure 1 has seven units. Table 2 presents different TAC at different values of EMAT for the two scenarios of using either the hot or the cold streams to define the intervals.

Table 2 TAC for IBMS approach.

Intervals	EMAT(K)	Units	Cost (\$ yr ⁻¹)
8 Hot	3	8	600,191
8 Hot	5.5	7	639,081
8 Hot	7.1	7	581,644
4 Cold	6.5	9	584,778
4 Cold	6.8	8	586,987
4 Cold	7	7	594,939

It is worth mentioning at this point that for the IBMS its only the MINLP model that needs to be solved for the optimal solution in a single step. This is unlike the stagewise superstructure of Yee and Grossmann (1990) in which the optimum configuration of the MINLP still needed an NLP suboptimisation step to be carried out in order to determine the optimal split stream flowrate ratio. The stagewise MINLP model of Yee and Grossmann (1990) gave nine units with three split streams. This configuration was further reduced to seven units in an NLP suboptimisation step with a TAC of \$576,640/yr, this is just 0.86% lower than the \$581,644 TAC of the IBMS which does not need an NLP suboptimisation step. However the network involves five splits all having isothermal mixing as illustrated by Figure 2.

Example problem 2

This example is taken from Shenoy et al. (1998). It involves two hot streams, two cold streams, three hot utilities (which comprise high, medium and low pressure steams) and two

cold utilities (cooling water and air cooling). Shenoy et al. (1998) applied the CUP and the results are compared with that of the IBMS in Table 3 below.

Table 3 :TAC for different designs and evolutions of CUP and the IBMS method

	CUP Target	CUP Design 1	CUP Evolution 1	CUP Design 2	CUP Evolution 2	IBMS
Area(m ²)	4956	5544	5046	5904	5636	5024
Units	-	11	9	11	9	9
TAC(\$/yr*10 ³)	1130.36	1182.94	1158.50	1212.69	1163.14	1148.88

The IBMS method gives a TAC of \$1148.88 ×10³/yr which is just 1.6% higher than the CUP target presented on Table 3. However the TAC of the IBMS is 2.9% and 5.26% lower than those of designs 1 and 2 of the CUP. Evolving the designs 1 and 2 of the CUP gives TAC presented on Table 3, these are still higher than the IBMS by 0.83% and 1.23% for evolutions 1 and 2 respectively. Note also that the IBMS does not need any special initialisations for the continuous variables.

4. Conclusion

A new robust IBMS model for the synthesis of HEN has been developed. This approach favours vertical heat transfer since the intervals are defined by the supply and target temperatures of either the hot or cold streams, hence split streams are automatically mixed at equal temperatures. Also the minimum number of units can be obtained. An outstanding feature of the IBMS is that no special initialisations for the continuous variables are needed. The heat capacity flowrate of the hot and cold utilities are only given initial points and lower bounds of 1 while arbitrary values which correspond to high values of heat recovery approach temperature (HRAT) are set for the upper bound.

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