CHEMICAL ENGINEERING TRANSACTIONS, Volume 12, 2007
Edited by Jiri Klemeš

Influence Of Geometric Parameters Of An Annular Fin Of Different Cross-Sections On Its Efficiency

Nabila Labsi and Youb Khaled Benkahla
Université des Sciences et de la Technologie Houari Boumediene USTHB,
Faculté de Génie Mécanique et de Génie des Procédés.
B.P. 32, El-Alia Bab-Ezzouar, 16111 Alger, ALGERIE.
e-mail: nabilalabsi@yahoo.fr , youbenkahla@yahoo.fr

Fins are used in various thermal devices such as industrial exchangers, evaporators, etc. The object of their use is to increase the heat transfer surfaces and therefore, the heat exchange. To contribute to the comprehension of heat transfer through these devices, a numerical study based on finite volume method is undertaken. The purpose of the present study is to examine, in the first step, the influence of geometric parameters of an annular fin of various cross-sections (rectangular, hyperbolic, triangular and parabolic) on its efficiency. These parameters are: its reduced length $\overline{R}$ and its characteristic parameter m_f. In the second step, the performances of the four cross-sections are compared. The validation of the computer code is confirmed by the comparison between the results obtained using the present model and those available in the literature, that is to say the analytical solution of Gardner and the numerical one of Ullman and Kalman.
Key words: annular fin, efficiency, finite volume method.

1. Introduction

Fins, in their various forms, are extended surfaces used in order to increase the total surface of exchange and therefore, to improve heat transfer. Gardner (1945) count among the first who studied various forms of fins. He presented the efficiency of longitudinal fin, annular one of rectangular and hyperbolic cross-sections and finally in form of spines. In a prospect for optimization of the profile and the dimensions of the annular fins, Ullman and Kalman (1989) presented abacuses giving the efficiency according to the characteristic parameter of annular fins of various cross-sections.

In the present study, annular fin of various cross-sections (rectangular, hyperbolic, triangular and parabolic) is considered. This fin is seat of an unsteady heat transfer following the radial direction. The purpose of this work is to determine the influence of the characteristic parameter as well as the reduced length of these four forms of fins, on the efficiency then to compare their thermal performances.

2. Formulation Of The Problem

A thermal balance on a differential element of length dr is carried out. The latter is taken within an annular fin of length L and variable thickness $\delta(r)$ fastened to a cylinder of radius r_0 and subject to a uniform temperature T_o at its base. Considering some simplifying assumptions (T_0 and T_∞ uniforms, very low Biot number and negligible heat flux evacuated at the extremity of the fin) and by introducing dimensionless variables x, $\bar{\delta}$, θ, τ, k_a, $\bar{R}$ as well as the characteristic parameter m_f of the fin and finally, by adopting the following laws to describe the variation according to the temperature of the thermal conductivity given by Zubair et al. (1996) as well as the heat transfer coefficient cited by Laor and Kalman (1996):

$$k(\phi) = k_0(1 + e\,\phi) \tag{1}$$

$$h = h_0 \phi^{v} \tag{2}$$

we lead to the following equation of energy:

$$x\,\bar{\delta}\frac{\partial \phi}{\partial \tau} = \frac{\partial}{\partial x}\left[k_a\, x\,\bar{\delta}\frac{\partial \phi}{\partial x}\right] - \left[\frac{m_f^2}{(\bar{R}-1)^2}\right]\left[\frac{1}{4}\left(\frac{\delta_0}{r_0}\right)^2\left(\frac{d\bar{\delta}}{dx}\right)^2 + 1\right]^{1/2} x\,\phi^{v+1} \tag{3}$$

The spatial and temporal boundary conditions are given below:

for $x = 1$ $\quad\phi(x,\tau) = 1$ (4)

for $x = \bar{R}$ $\quad\left.\dfrac{\partial \phi(x,\tau)}{\partial x}\right|_{\bar{R}} = 0$ (5)

for $\tau = 0$ $\quad\phi(x,\tau) = 1$ (6)

By considering all these variables constants and uniforms and that only x and τ constitute the independent variables, equation (3) can be solved numerically by means of finite volume method. By integrating the equation according to time and taking a linear profile of ϕ between the points w and e of the volume of control, one obtains:

$$\begin{aligned}\bar{\delta}_i\left(\frac{x_e^2 - x_w^2}{2}\right)\left(\phi_i - \phi_i^0\right) = &\left[k_{ae}\,x_e\,\bar{\delta}_e\left(\frac{\phi_{i+1}-\phi_i}{\delta x_e}\right) - k_{aw}\,x_w\,\bar{\delta}_w\left(\frac{\phi_i - \phi_{i-1}}{\delta x_w}\right)\right]\Delta\tau \\ &- \left[\frac{m_f^2}{(\bar{R}-1)^2}\left[\frac{1}{4}\left(\frac{\delta_0}{r_0}\right)^2\left(\frac{d\bar{\delta}}{dx}\right)_i^2 + 1\right]^{1/2}\left(\frac{x_e^2 - x_w^2}{2}\right)\phi_i^{v+1}\right]\Delta\tau\end{aligned} \tag{7}$$

In fact, the four forms of the studied fin differ by their dimensionless thickness $\bar{\delta}$ (Table 1). The latter is given by the following expression:

$$\overline{\delta} = \left(\frac{R_f - x}{R_f - 1} \right)^n \tag{8}$$

Table 1: Relation between *n*, R_f and the form of the cross-section of the annular fin.

Geometrical forms	*n*	R_f
Rectangular	0	$\overline{R}$
hyperbolic	-1	0
triangular	1	$\overline{R}$
parabolic	2	$\overline{R}$

A fin is often characterized by its efficiency which represents the quantity of heat leaving the base of the fin normalized by that evacuated if all the surface of the fin was maintained at T_0:

$$\eta = \frac{-\left(1 + e\phi_0\right) \left.\dfrac{\partial \phi}{\partial x}\right|_{x=1}}{\dfrac{m_f^2}{(\overline{R} - 1)^2} \phi_0^v \int_1^{\overline{R}} \left[\dfrac{n^2}{4} \left(\dfrac{\delta_0}{r_0} \right)^2 \dfrac{(R_a - x)^{2n-2}}{(R_a - 1)^{2n}} + 1 \right]^{1/2} x \, dx} \tag{9}$$

It should be noted that the results presented on this paper do not take into account the variability according to the temperature of thermal conductivity and heat transfer coefficient. Actually, the parameter e and the exponent v will be taken equal to zero.

3. Results And Discussion

3.1 Validation of the computer code

To validate our computing program, a comparison between the present results and those of the literature was made; it is about the numerical solution of Ullman and Kalman (1989). The latter study considers an annular fin of a rectangular cross-section for which all the physical properties as well as temperatures T_0 and T_∞ are constant and uniform. The heat flow is unidirectional and takes place in steady state. Besides, the fin whose extremity is isolated, does not generate internal heat.

The comparison shows a good agreement between our results and those of the literature. *Figure* 1 shows indeed an almost perfect concordance between the numerical solution of Ullman and Kalman (1989) and the present model, except that a light shift for the low values of the parameter m_f, to which a difference of 4% is noted.

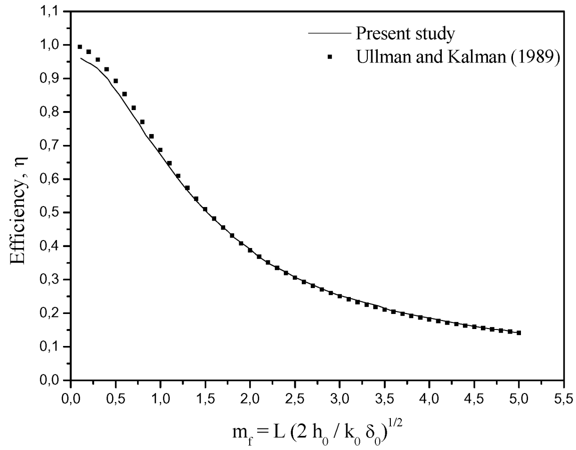

Figure 1: Comparison between the efficiency resulting from the present study and those of Ullman and Kalman. Rectangular fin, $(\delta_0/r_0) = 0.01$ and $\bar{R} = 2$.

2.2 Effect of the characteristic parameter and the reduced length of the studied fins on the efficiency

Figure 2 shows the evolution of the efficiency for the four fin's profiles according to their characteristic parameter m_f , for $\bar{R}$ = 2, 3, 4 and 5 when the steady regime is reached (τ = 0.8). The curves present the same form: a decreasing monotonous function of the parameter m_f ranging between the values 1 and 0, tending asymptotically towards a plate when m_f tends to 5. The maximum values of the efficiency are at the low values of m_f. We notice that whatever the value of m_f, the efficiency decreases when $\bar{R}$ increases. This decrease is very pronounced for the low values of m_f, then, more and more slow for the great ones. Because of the low lengths of the fin (for δ_0, k_0 and h_0 given) and/or the great values of the thermal conductivity of the material (for L, δ_0 and h_0 given), the real heat flux dissipated by the fin becomes indeed equal to the maximum heat flux which could be dissipated by the fin.

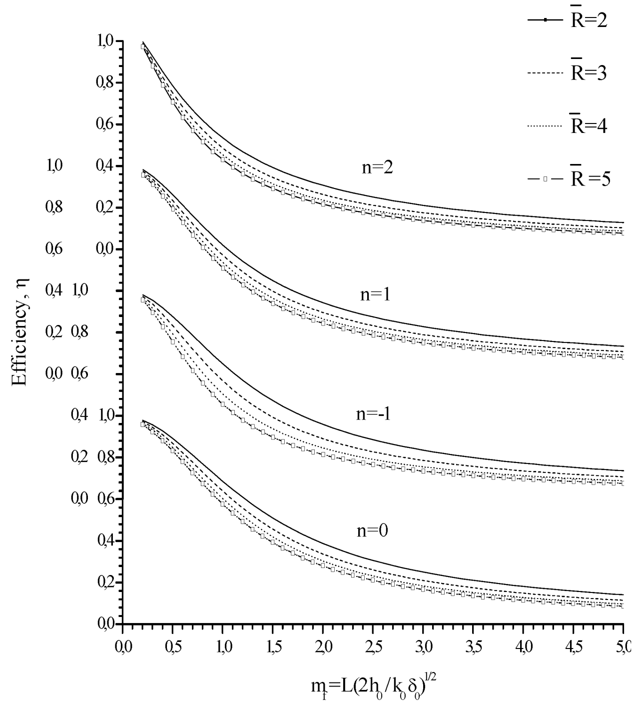

Figure 2: Evolution of the efficiency of an annular fin of various cross-sections according to the characteristic parameter m_f, for various values of $\bar{R}$.. τ = 0.8.

3.3 Comparison between the thermal performances of the studied fins

Figures 3-a and 3-b show the variation of efficiency of the studied annular fins according to the characteristic parameter m_f after the reach of the steady regime (τ =0.8) and for $\bar{R}$ equal to 2 and 5, respectively. The comparison between the fins in term of thermal performances, shows that the rectangular fin possesses, whatever the value of m_f, the greatest efficiency. This observation is raised by several authors.

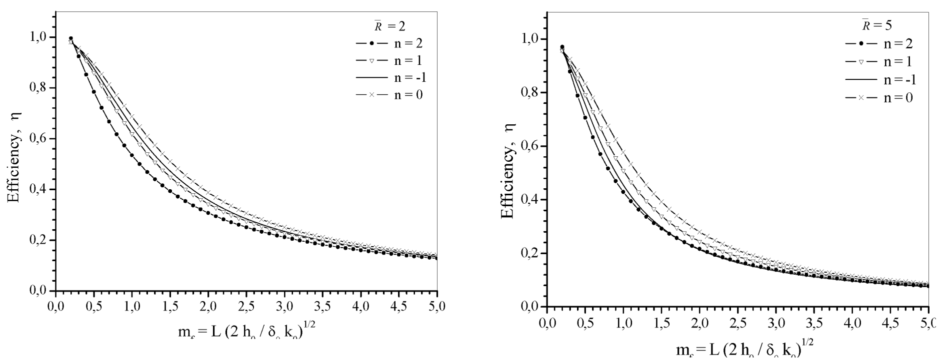

Figure 3: Evolution of the efficiency according to the characteristic parameter m_f for various cross-sections of the studied fin τ = 0.8. (a) $\bar{R}$ = 2 , (b) $\bar{R}$ = 5.

It is also interesting to note that for $\bar{R} = 5$ the hyperbolic and parabolic profiles merge for values of m_f higher than 1.2. Similarly, for $\bar{R} = 2$, the hyperbolic and triangular profiles merge almost for $m_f \geq 2$. On the other hand, while going from $\bar{R} = 2$ to $\bar{R} = 5$, the triangular fin becomes more efficient than the hyperbolic one.

4. Conclusion

The purpose of the present study was to determine the influence of the geometrical parameters (m_f and $\bar{R}$) of an annular fin of various cross-sections on the efficiency by considering constant physical properties. The performance of the computing program was confirmed by comparing the present results with the data of the literature. We note that the maximum values of the efficiency are obtained for the low values of the characteristic parameter of the fin and for the low values of $\bar{R}$. In addition, whatever the conditions, the annular fin of rectangular cross-section ($n = 0$) possesses the best values of efficiency in comparison to the three other geometrical forms.

5. References

Gardner K.A., 1945, Efficiency of extended surfaces, ASME J. Heat Transfer, 67.

Laor K. and H. Kalman, 1996, Performance and optimum dimensions of different cooling fins with a temperature dependent heat transfer coefficient, Int. J. Heat and Mass Transfer, 39, pp 1993-2003.

Ullman A. and H. Kalman, 1989, Efficiency and optimized dimension of annular fins of different cross-section shapes, Int. J. Heat and Mass Transfer, 32, pp 1105-1110.

Zubair S.M., A.Z. Al-Gani and J.S. Nizami, 1996, The optimal dimensions of circular fins with variable profile and temperature-dependent thermal conductivity, Int. J. Heat and Mass Transfer, 39, pp 3431-3439.

5. Nomenclature

- C_p specific heat of the fin material
- h_0 constant term in the coefficient of heat exchange
- h coefficient of convection heat exchange
- k thermal conductivity of the material
- k_0 thermal conductivity taken at 0 °C
- k_a dimensionless thermal conductivity, $= k(T) / k_0$
- L length of the annular fin
- m_f characteristic parameter of the fin, $= L\,(2\,h_0 / k_0\,\delta_0)^{1/2}$
- n parameter which describes the variation of the profile of the fin
- r radial position
- r_0 radius of the cylindrical tube
- r_1 radial position of the extremity of the fin, $= r_0 + L$
- $\bar{R}$ dimensionlessal exterior radius, $= r_1 / r_0$
- R_f parameter which depends on the form of the fin profile
- t time
- T temperature
- T_0 temperature at the base of the fin
- T_∞ surrounding temperature
- x dimensionless radial coordinate, $= r / r_0$

Greek symbols:

- $\delta(r)$ thickness of the fin at a radial distance r
- $\bar{\delta}$ dimensionless thickness, $= \delta(r)/ \delta_0$
- η efficiency of the annular fin
- ρ density of the fin
- ϕ dimensionless temperature, $= (T - T_\infty)/(T_0 - T_\infty)$
- τ dimensionless time, $= k_0\, t / \rho\, C_p\, r_0^2$

Subscripts

- e, w control volumes faces at east and west sides
- i position of the node