CHEMICAL ENGINEERING TRANSACTIONS, Volume 12, 2007
Edited by Jiri Klemeš

Secondary Flow Effects in a Trapezoidal Isothermal Minichannel Flow System Involving a Sharp-Edged 180^o Bend

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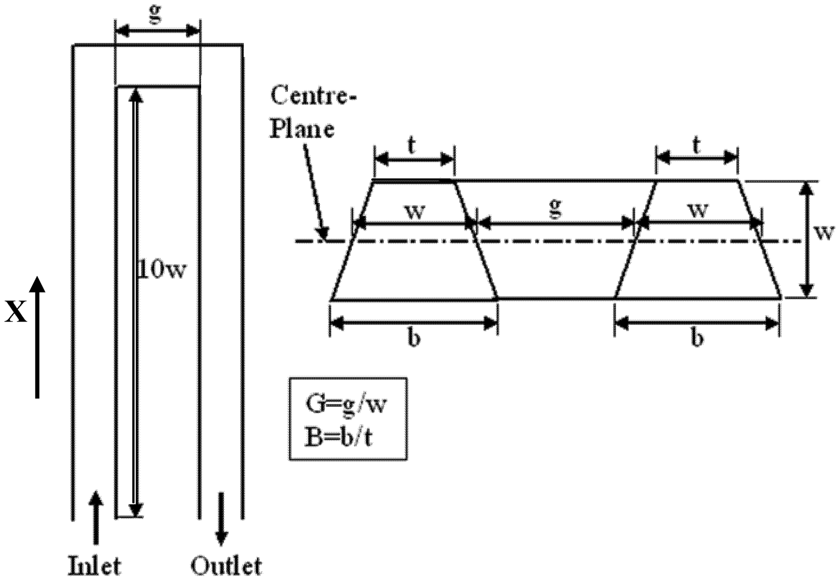
When the flow in a minichannel system passes around a bend, secondary flows develop and flow separation can also occur in the bend both of which can affect the flow, pressure drop and heat transfer rate in the bend and in the flow upstream and downstream of the bend. In order to investigate this, attention has been given to flow in a simple such system. In the system considered the channels have a trapezoidal cross-section with parallel top and bottom surfaces. The flow enters the channel with a uniform velocity and temperature and passes through a straight channel that has a length that is 10 times the size of the channel. The flow then passes around a sharp 90^o bend, flows through a short straight channel and then flows through another sharp 90^o bend. Following the two bends, the flow passes down another straight channel which also has a length 10 times the size of the channel. A uniform temperature exists over the entire surface of the channel. It has been assumed that the flow is steady and incompressible and that there is no slip at the channel walls. The governing equations have been written in dimensionless form. Solutions to these dimensionless governing equations have been obtained using a commercial finite-element software package, FIDAP. The solution has the following parameters: the Reynolds number, the Prandtl number, the dimensionless length of the straight pipe section between the two bends and the ratio of the bottom-to-top widths of the channel. Results have been obtained for a Prandtl number 0.7 for Reynolds numbers of between 100 and 5000 for various values of the other dimensionless parameters.

1. Introduction

Minichannels are here assumed to be channels with a size that is of the order of 1 mm and with flows through them that are such that the Reynolds number is between about 1 and 1000. Such channels are used in some fuel cells and in some electronic cooling systems. In some such applications the channels have a trapezoidal cross-section with parallel top and bottom surfaces. When the flow in such a channel passes around a bend, secondary flows develop and flow separation can also occur in the bend which can affect the pressure drop and heat transfer rate in the channel system. In order to investigate this, attention has been given to flow in a simple such minichannel system. In the system considered, the flow enters the channel with a uniform velocity and temperature and passes through a straight channel that has a length that is 10 times the size of the channel. The flow then passes around a sharp 90^o bend, flows through a short straight channel and then flows through another sharp 90^o bend. Following the two bends, the flow passes down another straight channel which also has a length of 10 times the size of the channel. The flow system considered is therefore as shown in Fig. 1. A uniform temperature exists over the entire surface of the channel. This flow geometry does not represent that which occurs in any applications but is adequate for the evaluation of the effects of secondary flow and flow separation. Pressure losses and



Centre-Plane Section
Through Channel System

Figure 1: Channel System Considered

heat transfer in duct flows have been quite widely investigated (e.g. see Gamrat et. al. (2004)). The effects of channel cross-sectional shape on microchannel duct flows are discussed, for example, by Wu and Cheng (2003) and Oosthuizen (2006). These studies do not specifically look at the effect of the secondary flow that develops in a bend in the channel on the characteristics of the flow. The major influence of a rounded bend on the flow in a channel system is due to the formation of Dean vortices whose axes are aligned with the flow direction. In the case of a rounded bend these vortices produce an enhancement of the heat transfer rate relative to the rate in similar flows in straight pipes, see for example, Ligrani et. al. (1997), and Chintada et. al. (1999). Most of these existing studies of low Reynolds number flows in which Dean vortices have an effect have considered channels with square or rectangular channel cross-sectional shapes, for example see Yamamoto et al. (2004), Yanase et al. [(2002), Chandratilleke and Nursubyakto (2003), and Yanase et al. (2005). The main purpose of the present work was to see whether the use of a trapezoidal channel shape will have a significant influence on the results. Also when a sharp bend is used there is flow separation in the bend. This is expected to influence the Dean vortex formation and, therefore, to influence the effects that these vortices have on the flow. This is examined here.

2. Solution Procedure

It has been assumed that:

1. the flow is steady,
2. the flow is incompressible,
3. the velocity is uniform over the channel inlet plane,

4. there is no slip on the boundaries,
5. the channel walls are maintained at a uniform temperature.

The governing equations have been written in terms of dimensionless variables based on the use of the mean channel width w (see Fig. 1) as the length scale and the mean velocity u_m , which is equal to the uniform inlet velocity because incompressible flow is being assumed, as the velocity scale. The following dimensionless pressure has been defined as $P=(p-p_0)/\rho u_m^2$ where p_0 is the pressure on the channel outlet plane. The governing equations written in terms of these dimensionless variables have been solved using the commercial finite-element software package, FIDAP. The solution gives the variations of dimensionless temperature and dimensionless velocity across the channel cross-section. These results can then be used to find the mean fluid temperature and mean circumferential wall heat flux at any section along the channel, i.e., at any value of X. Having established the values of the mean fluid temperature and of the circumferentially averaged wall heat flux at a given position in the flow along the channel, the mean Nusselt number, Nu , at any position along the channel can then be found using $Nu = q_w w / k(T_w - T_m)$.

3. Results

The solution has the following parameters:

1. the Reynolds number, *Re*
2. the Prandtl number, *Pr*
3. the bottom-to-top width ratio (see Fig. 1) of the channel cross-section, $B = b/t$
4. the dimensionless distance between the inlet and outlet channels (see Fig. 1), $G = g/w$.

Because of the possible applications that motivated the study, results have only been obtained for $Pr = 0.7$. *Re* values between 100 and 1000, B values of 1.0, 1.5 and 2.0 and G values of 0.5, 1.0 and 2.0 will be considered here. Typical velocity distributions over the cross-section of the channel at a dimensionless distance of 2 before and after the bend in the inlet and outlet channels respectively are shown in Fig. 2. The results given in Fig. 2 illustrate the effect of B on the distributions. The secondary flow and low velocities near the inner wall in the flow downstream of the bend are clearly illustrated by the results given in Fig. 2. The profiles upstream of the bend indicate that the effects of the bend on the upstream flow are relatively weak except for the region of the channel that lies just upstream of the bend. The added distortion to the velocity distribution caused by increasing the value of B is also shown by the results.

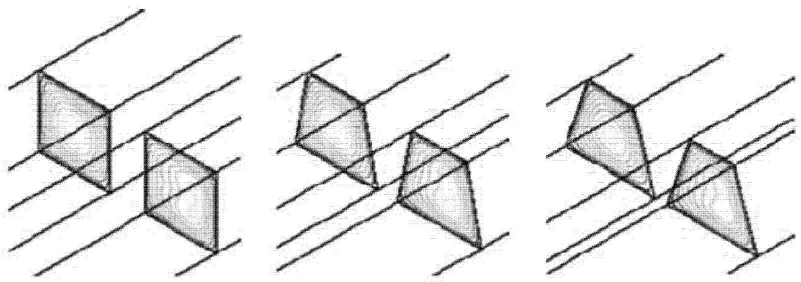


Figure 2: Effect of Channel Bottom-to-Top Ratio, B, on the Variation of Dimensionless Flow Speed Over Channel Sections in the Inlet (left) and Outlet (right) Channels at $X = 8$. Results are for B=1 (top), B=1.5 (middle) and B=2 (bottom) and for Re=100 and G=0.5.

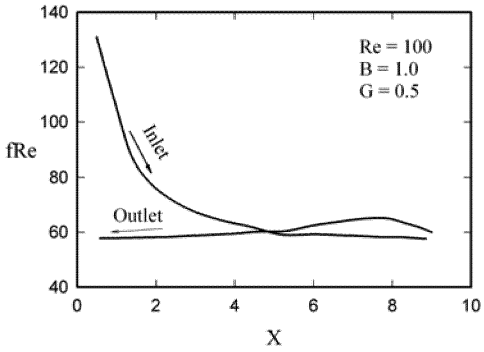


Figure 3: Variation of fRe with X in Inlet and Outlet Channels for Re =100, B=1 and G=0.5.

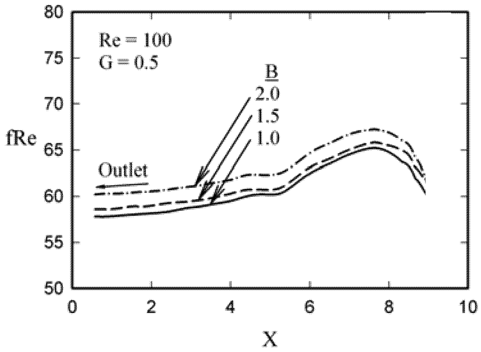


Figure 4: Variation of fRe with X in Outlet Channel for Re =100 and G=0.5 for Various Values of B.

Attention will next be turned to the pressure variation along the inlet and outlet channel sections. In fully-developed flow the pressure is usually expressed in terms of the friction factor, *f*, which, in terms of the dimensionless variables being used here, is given by $-dP/dX = f/2$. Now in fully developed laminar flow it is usually adequate to assume that $f = K/Re$, i.e., that $f\,Re = K$ where *K* is a constant. Combining the equations then gives $f\,Re = -2\,Re\left(dP/dX\right)$. It should be noted that these equations have been written in terms of *w* rather than the hydraulic diameter. However, for the range of *B* values considered here the difference between *w* and the hydraulic diameter is small. The variation of *P* across a section of the duct is relatively small even when the secondary flow exists so the pressure gradient can be taken as the gradient on the duct centre-line. Typical variations of *fRe* with dimensionless distance, *X*, along the channel for both the inlet and outlet channels are shown in Fig. 3. It will be seen from this figure that in the flow development region at the smaller values of *X* in the inlet channel and at the larger values of *X* in the outlet channel flows the value of *f Re* is not constant. At the larger values of *X* in the inlet channel and the smaller values of *X* in the outlet channel, however, as fully-developed flow is approached, *fRe* does approach a constant value. Now for fully-developed flow classical analysis gives the value of *fRe* for flow in a square channel as 56.93 and it will be seen that the values of *fRe* do approach a value of similar magnitude near the ends of the channels. It will also be seen from the results given in Fig. 3 that the main effect of the bend is on the flow in the outlet channel. Attention will therefore be concentrated on the results for the outlet channel. Figures 4 and 5 show variations of *fRe* with *X* in the outlet channel for various values of *B* for *G* = 0.5 and for *Re* = 100 (Fig. 4) and *Re* = 300 (Fig. 5). The effect of *B* on the general form of the variation at the lower Reynolds number considered will be seen to be relatively small. However at the higher Reynolds number considered due to the increased strength of the secondary flow and the increased extent of the flow separation in and near the bend *B* has a significant effect on the distribution just downstream of the bend. Consideration will next be turned to the Nusselt number distribution. The variation of *Nu* with *X* in the inlet channel was found to essentially depend only on the Reynolds number. The distribution shown in Fig. 6, for example, applies for all the values of *B* and G considered here. It will be seen from the results given in Fig. 6 that the presence of the bend leads to a decrease in the Nusselt number value in the region of the channel that is just downstream of the bend.

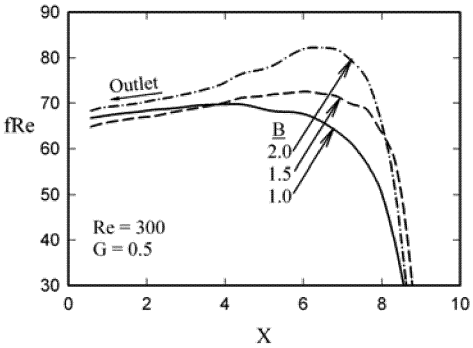


Figure 5: Variation of fRe with X in Outlet Channel for Re =300 and G=0.5 for Various Values of B.

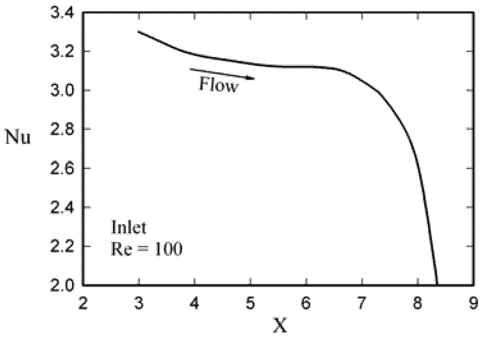


Figure 6: Variation of Nusselt Number with X in Inlet Channel for Re =100..

Attention will next be turned to the *Nu* variation in the outlet channel. Figures 7 and 8 respectively illustrate the effects of *G* and *B* on this distribution. It will be seen from these figures that the secondary flow generated in the bend and the flow separation in the bend lead to a decrease in the Nusselt number just downstream of the bend but *Nu* then increases to a value above that which applies in fully-developed channel flow. It will also be seen from Figs. 7 and 8 that *G* has a relatively small effect on the *Nu* variation but that at a particular value of *X, Nu* decreases significantly with increasing *B*.

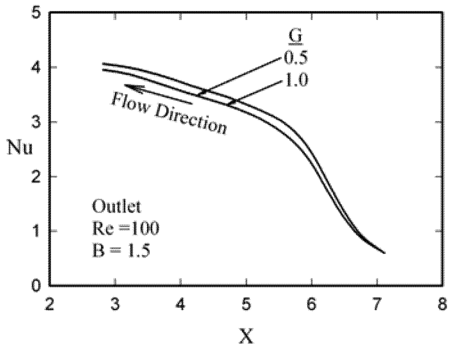


Figure 7: *Variation of Nusselt Number with X in Outlet Channel for Re =100 and B=1.5 for two value of G.*

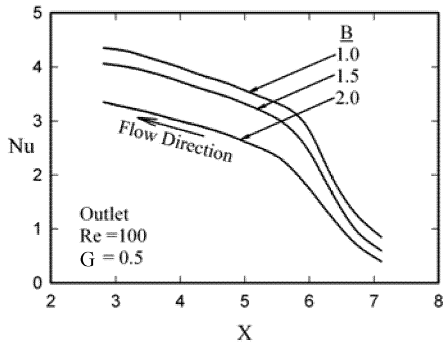


Figure 8: *Variation of Nusselt Number with X in Outlet Channel for Re =100 and G=0.5 for three value of B.*

4. Conclusions

The results obtained in the present study show that the presence of the secondary flow generated in the bend and of the flow separation in the bend there are, in general, changes in the flow both upstream and downstream of the bend. However, the changes in the outlet duct downstream of the bend are greater than those in the inlet duct upstream of the bend. In the outlet duct there are significant decreases in both *fRe* and *Nu* just downstream of the bend. However, with increasing distance from the bend *Nu* then rises significantly above the fully-developed value. For the duct lengths and conditions considered here, fully-developed flow is not reached in the outlet duct. The results also indicate that for the type of bend here considered the effect of the dimensionless bend width *G* on the variations of *fRe* and *Nu* is small but that the channel cross-sectional shape parameter *B* can have an important influence on the variations of *fRe* and *Nu* in the outlet channel.

5. Acknowledgements

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

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