# The Sequential Framework For Heat Exchanger Network Synthesis – Network Generation And Optimization

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A Sequential Framework for Heat Exchanger Network Synthesis (HENS) is presented. The network generation and optimization phase of the framework, one of its core subproblems, is presented with details as to the source of the non-convexities in the model. Physical insight based automated starting value generators are developed to ensure that the base NLP formulation solves to a 'good' local optimum. Two methods of dealing with the non-convexities are also briefly presented.

## 1. Introduction

The Heat Exchanger Network Synthesis (HENS) problem involves solving a three-way trade-off between energy (E), heat transfer area (A), and how this total area is distributed into a number of heat transfer units (U). For details about the subject, see exhaustive reviews by Gundersen and Naess (1988) and Furman and Sahinidis (2002). Optimization methods have been routinely applied in an effort to solve the complex and multiple trade-offs that are inherent to the HENS problem. Simultaneous MINLP models (for example Yee and Grossmann, 1990) can, in theory, address and solve the trade-offs in the HENS problem. These models, however, have demonstrated severe numerical problems related to the non-linear (non-convex) and discrete (combinatorial) nature of the HENS problem. Even with the rapid advancements in computing power and optimization technology, the size of the problems solved with these methods does not meet industrial needs.

The HENS problem has been proven to be NP-hard in the strong sense by Furman and Sahinidis (2001) and has prompted a renewed interest in synthesis methods for HENS that utilize the strategy of dividing the HENS problem into a series of sub-problems to reduce the computational complexity of obtaining a network design. This paper presents developments in the network generation and optimization step of a Sequential Framework described below.

# 2. The Sequential Framework

As a compromise between Pinch Analysis and simultaneous MINLP models, a sequential and iterative framework has been in development in our group with the main objective of finding near optimal heat exchanger networks for industrial size problems.

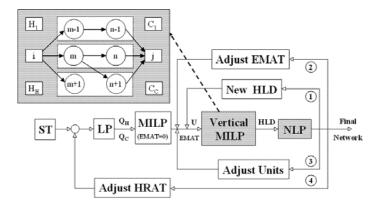


Figure 1: The Sequential Framework for HENS with the Vertical Transportation Model

The subtasks of the design process are solved sequentially using Math Programming. Briefly, these steps involve: establishing the minimum energy consumption (LP), determining the minimum number of units (MILP), finding sets of matches and corresponding heat load distributions (HLDs) for minimum or a given number of units (MILP), and network generation and optimization (NLP) as shown in Figure 1.

The Sequential Framework is based on the recognition that the selection of HLDs impacts both the quantitative (network cost) and the qualitative aspects such as network complexity, operability and controllability. The Vertical MILP model for selection of matches and the subsequent NLP model for generating and optimizing the network form the *core engine* of the framework.

Significant user interaction is built into the framework in the form of iterative loops to enable the designer to explore and evaluate the most promising networks with respect to Total Annual Cost (TAC), network complexity (number of units, splits, etc.), operability and controllability.

# 3. Network Generation and Optimization

In the Sequential Framework, network generation and optimization is performed by an NLP formulation, where the actual network topologies are extracted from the stream superstructure given in Floudas et al. (1985). All possible network structures for a given set of HLDs are included in this superstructure. The objective function is the minimum total investment cost for the heat exchangers, since the inner loop of the Sequential Framework is explored with fixed level of heat recovery and hence fixed operating cost. The NLP formulation is non-convex and the sources of non-convexity are detailed in section 3.2. As indicated earlier, the NLP is part of the core engine of the framework and hence a solution to this sub-problem is essential. Earlier work (Anantharaman and Gundersen, 2006) explored a novel modal trimming approach to solving the problem. This paper presents two, more conventional, approaches (steps) to solving the non-convex NLP.

### 3.1 Multiple Starting Points

For the numerical solution of the NLP formulation, it is important to start with a "good" initial guess for deriving the network configuration. This is particularly true for large industrial sized problems where a good initial guess is a prerequisite for getting a solution, not to mention a globally optimum one. Multiple starting points allow the user to explore the solution space, and in the case of a difficult problem, ensure a feasible solution. This section details five automated starting value generators developed for this NLP formulation in an Excel/GAMS environment for Sequential Framework called SeqHENS. The guiding light has been to use physical insight to ensure "good" local optima. The starting value generators are described and then summarized as regards their efficacy in solving a set of 5 test cases (10 HLDs) from the literature.

Heat capacity flow rates and temperatures of the streams in the superstructure are the optimizing variables, with heat capacity flow rates being identified as the decision variables that are in turn used to calculate the temperatures. Thus, the starting value generators will mainly involve setting the heat capacity flow rates.

#### 3.1.1 Basic Serial/Parallel Generator

This is a simple and very flexible method of setting the starting values. For each stream, the user decides if the stream configuration should be pure serial or parallel. In case of serial configuration, the user has a further choice regarding the sequence of matches as shown in Figure 2. This method is not based on physical insight but provides the user with a great deal of flexibility and hence a large number of starting values.

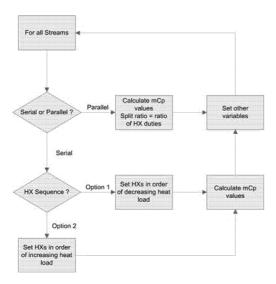


Figure 2: Serial/Parallel starting value generator

#### 3.1.2 Serial H/H Heuristic Generator

As the name suggests, this starting value generator is based on the hottest/highest heuristic proposed by Ponton and Donaldson (1974). For a given set of matches  $(i, j) \in MA$  for a hot stream i, the hot supply end of the stream is matched with a ranked set of cold stream matches such that cold stream j with  $max(T_j^{OUT})$  is matched with hot stream i at the hot supply end. This generator includes physical insight in the use of temperature driving forces but does not consider the match duty and stream heat capacity flow rates.

# 3.1.3 VertMILP based Generator

This starting value generator uses results from the VertMILP model that generates the HLDs. The temperature range  $(\Delta T)$  for the hot and cold streams of each match  $(i,j) \in MA$  is available from the VertMILP model and the starting value generator tries to replicate this temperature range in the stream superstructure.

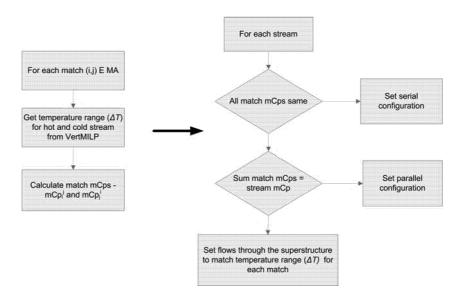


Figure 3: VertMILP based generator

This method is based on the initialization procedure described in Floudas et al. (1986), and has been modified based on the fact that there is no pinch decomposition in the Sequential Framework.

# 3.1.4 Combinatorial Generator

The combinatorial generator utilizes all stream and match information to generate a feasible network as the starting point. The first step in this method is to allocate the utilities – they are set to match with process streams at their target end and a new modified target temperature is calculated. The next step is to check streams with 1

match for feasibility with all other streams - exchange at the modified target end of the process streams is only permitted. This ensures that the possibility of feasible exchanges increases as we proceed. The next step is to check for feasible exchanges for hot streams with multiple matches. This is also done similar to the earlier cases where a match is allowed only at the target end of the cold stream. Once all hot streams are done, the procedure involves looping through these steps (checking feasibility of all single match streams and hot multiple match streams), as opportunities may have opened up for fixing exchangers, until there are no more exchangers that can be fixed. The remaining matches are set up, using split streams, as parallel exchangers where the splits are calculated based on the temperature range and match duty.

This procedure is time consuming, but ensures a feasible starting point.

#### 3.1.5 Results

The starting value generators were tested on 5 literature problems ranging in size from 4 to 15 process streams and 2 utilities. The combinatorial generator always ensured that an optimum was found and this optimum was the lowest compared to runs with other starting values. The VertMILP based generator and the parallel generator performed second best and ensured that a feasible solution was found in 90% of the cases, with the parallel configuration performing better for larger problems. The serial H/H heuristic generator produced a feasible solution in 50% of the cases, while the pure serial configuration gave a feasible solution in only 10% of the cases. Thus, the results show a considerable difference in the performance of the various starting value generators.

#### 3.2 Solving the non-convex NLP for Global Optimum

The NLP model involves the following sources of non-convexities that may result in local optima:

- 1. Products of variable flowrates and temperatures in the heat balances for mixers and exchangers.
- 2. Equations that define the log mean temperature differences used to calculate heat transfer area.
- 3. The economy of scale type cost equation that relates heat transfer area to investment cost.

It is important to notice, however, that there is one major difference between the Sequential Framework and the simultaneous MINLP methods. One of the true advantages of the Sequential Framework is the fact that heat duties of the matches are given by the MILP model and thus fixed when solving the NLP model. As a result, the objective function for the NLP turns out to be convex. Thus the supposed nonconvexities 2 and 3 vanish in the Sequential Framework.

Floudas et al. (1989) present an approach to global optimization of non-convex NLP and MINLP problems based on Benders Decomposition. The variables set is decomposed into two sets – complicating and non-complicating variables – resulting in the decomposition of the constraint set leading to two subproblems. A series of these subproblems are solved to determine the global optimum. They present a graph theoretical approach to determining the various possibilities of decomposing the variable set.

The heat capacity flow rates are chosen as the complicating variables for the NLP formulation under consideration. The starting point required for this procedure is taken to be the 'local optimum' found from the methods described in Section 3.1.

Another approach is the use of convex relaxations for the bilinear terms that cause the non-convexities. Hashemi-Ahmady et al. (1999) describe such relaxations that can be used in conjunction with the Sequential Framework.

The former approach is the more desirable method as it mainly involves solving a set of linear problems. It is less computationally expensive than the latter method, but is certainly more expensive than the basic NLP formulation.

# 4. Conclusion and further work

The network generation and optimization phase of the Sequential Framework, one of its core subproblems, is presented with details as to the source of the non-convexities in the model. It is seen from the discussion that the NLP formulation in the Sequential Framework is much easier to solve than the MINLP formulations for HENS since the non-convexities involved are substantially reduced. Non-convexity is still an issue for this NLP formulation. Automated starting value generators based on physical insight are developed to ensure that the base NLP formulation solves to a 'good' local optimum. Two methods of dealing with the non-convexities are briefly presented. Both these methods require a good starting point, which is provided by the optimum found using the starting value generators. Future work will involve exploring the use of Simulated Annealing to solving the NLP formulation to global optimum.

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